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Measuring the natural rate of interest in a commodity exporting economy: Evidence from Mongolia

Gan-Ochir Doojav [a,*], Munkhbayar Gantumur [b,1]

[a] Bank of Mongolia, Baga Toiruu-3, 15160, Ulaanbaatar 46, Mongolia
[b] Research and Statistics Department, Bank of Mongolia, Baga Toiruu-3, 15160, Ulaanbaatar 46, Mongolia

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ABSTRACT

This paper examines the fundamental drivers of the natural rate of interest in a commodity-exporting economy (Mongolia) based on a structural New Keynesian model that incorporates external factors such as demand for commodity exports, commodity price, and FDI, using Bayesian techniques. Our empirical analysis resulted in the following novel findings. First, we find evidence of transitory shocks to the real natural rate of interest. Second, we show that the estimated real natural rate of interest is procyclical. It declined during 2012–2016 and has been gradually recovering since 2017. The monetary policy stance has been neutral since mid-2017. Third, due to the characteristics of the Mongolian economy, the natural rate of interest remains at double-digit, thus explains high interest rate in the country. Fourth, our findings highlight that external shocks (i.e., spillovers from the global commodity market) are the main determinants of the real natural rate of interest, as they account for 40 percent of its fluctuations. Consequently, the external shocks mainly transmit through the equilibrium real exchange rate.

1. Introduction

Interest rate has played a central role in macroeconomic and monetary theory since Wicksell (1936), Keynes (1936), and Hayek (1939). Wicksell (1936) characterized the natural rate of interest as a certain rate of interest on loans that is not likely to encourage change in prices in either direction. Consistent with the notion, in the literature the natural rate of interest is defined as the real short-term interest rate consistent with natural level of output and stable inflation. In modern days, most central banks use the short-term interest rate as their key monetary policy instrument. By controlling the short-term interest rate, they are able to influence the real interest rate that drives agents' economic decisions. Since it is difficult to determine whether a certain level of real interest rate is expansionary or contractionary, real-time estimation of the natural rate provides a benchmark for assessing the monetary policy stance.

Literature on estimating the natural rate of interest has been revisited often since the recent Global Financial Crisis (GFC), given the persistently low interest rates in advanced economies. In addition to measuring international trends of natural rate of interest, recent literature (i.e., Holston et al., 2017; Fries et al., 2018, and Del Negro et al., 2019) is also interested in investigating the determinants of the natural rate. Few papers (Us, 2018 for Turkey, Wang, 2019 for China, Dash and Bhole, 2017 for India) have estimated time-varying

* Corresponding author.
E-mail addresses: doojav_ganochir@mongolbank.mn (G.-O. Doojav), munkhbayar.g@mongolbank.mn (M. Gantumur).
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natural rate of interest for emerging economies, however they used structural models in a closed-economy setting. Only recently, empirical studies (i.e., Berger and Kempa, 2014, Wynne and Zhang, 2018a; Hlédik and Vlček, 2018a, b, Grossman et al., 2019) have explicitly focused on small open economies and spillovers from the rest of the world (i.e., strong impact of foreign output shocks on the natural rate). Though there are some structural models designed for estimating the natural rate of interest in small open economies, none of them specifically focuses on commodity dependent economies. Moreover, existing literature does not explicitly investigate contribution of external shocks such as global commodity price, commodity demand, and foreign direct investment (FDI), to the natural rate in small open economies. Identifying the open-economy determinants is an important step to understand movements of the natural rate in line with the global outlook and to shape domestic policy responses.

In the context, this paper examines the fundamental drivers of the natural rate of interest in a commodity exporting economy using a structural New Keynesian model that explicitly incorporates demand for commodity exports, commodity prices, and FDI. Our small open economy modeling framework is in line with some closely related papers (i.e., Berger and Kempa, 2014; Wynne and Zhang, 2018a; Grossman et al., 2019), and deviating from them, we introduce (i) external factors into the baseline model of Berg et al. (2006) to account for the fact that commodity exporting economies are frequently hit by shocks of commodity price, demand for commodity exports, and FDI, and (ii) equilibrium real output growth adjusted for equilibrium real exchange rate appreciation into the natural rate equation of the model to account for the open economy dimension. Consistent with the recent literature (i.e., Holston et al., 2017, Del Negro et al., 2019), we jointly estimate the natural rate of interest, inflation target, output gap, potential output growth, and equilibrium real exchange rate within our reference model using Bayesian techniques. This paper widens the existing literature since it is among one of the first attempts to investigate the impact of external shocks (i.e., spillovers from the global commodity market) to the natural rate in a commodity exporting economy with a structural small open economy model. Our paper establishes a novel finding that shocks of commodity price, demand for commodity exports, and FDI play an important role in the determination of the natural role in a commodity exporting economy.

The empirical analysis is applied to Mongolia, a commodity dependent economy which is puzzled by high interest rate.[2] For instance, more than 90 percent of its exports value is mineral based, while production of the resource sector accounts for about 20 percent of gross domestic product (GDP) over the last decade. As a consequence, business cycle fluctuations are highly prone to external shocks such as sudden flood or stop of FDI, and changes in commodity demand and commodity prices (Gan-Ochir and Davaajargal, 2019). Another motivation to estimate the natural rate of interest for Mongolia is to identify whether or not the economy's high dependence on commodities is a key driver of high interest rates in Mongolia. Over the sample period, average headline annual inflation was 9 percent, nominal policy rate was 12 percent ranging between 10 and 15 percent, and (weighted average) annual bank lending rate was 19.1 percent fluctuating between 16.9 and 23.5 percent in Mongolia. Hence, evidence from the Mongolian case would be of high relevance to measure the natural rate, identify foreign determinants of high interest rates, and design monetary policy in commodity exporting small open economies.

Over the last two decades, three approaches have been actively used in estimating time-varying natural rate of interest: demographic trends in OLG models (i.e., Bielecki et al., 2018; Papetti, 2019), econometric approach and dynamics stochastic general equilibrium (DSGE) models (i.e., Gerali and Neri, 2019; Haavio et al., 2017). The econometric approach is also divided into two sub-approaches such as VAR-based approach (i.e., Brzoza-Brzezina, 2002; Lubik and Matthes, 2015; Jarocinski, 2017) and semi-structural econometric model/variants of Laubach and Williams (2003) approach.

In this paper, we rely on the semi-structural econometric approach, thus focus on the relevant literature review. The approach is developed by Laubach and Williams (2003) who use Euler equation to assess the connection between the unobserved natural rate of interest and the annualized trend in growth of potential output. Using the United States data, they find a close link between the natural rate of interest and the trend in growth rate, and significant variations were observed in time-varying natural rate of interest over the period of 1961–2002. Based on the Laubach-Williams methodology, Holston et al. (2017) detected large decline in the trend of GDP growth and natural rates of interest over the last three decades in advanced economies (the United States, Canada, the Euro Area and the United Kingdom). They also found a substantial degree of co-movement among the natural rates of all four economies over time, suggesting a vital role of global factors in shaping the trend in growth and natural rate of interest. By applying the same methodology, Wynne and Zhang (2018b) estimate the time-varying natural rate for the world as a whole and find that the world natural rate has been on a declining trend for the past few decades. Laubach and Williams (2018) find that the estimated natural rate of interest for the United States has declined dramatically since the start of the Great Recession.

The literature on estimating the natural rate of interest in small open economies is rather recent. Berger and Kempa (2014) develop an open economy version of the Laubach and Williams (2003) model and estimate it for Canada. In their model, both aggregate demand the Phillips curve contain the real exchange rate as an argument and is further extended by an equation linking the real interest rate to the real exchange rate. The same model is estimated by Pedersen (2015) for Denmark. Wynne and Zhang (2018a) estimate the natural rate of interest for the United States and Japan with an open economy variant of the model proposed by Laubach and Williams (2003) and study international interaction (through trend in growth rate of foreign economy) in determining the natural rate of interest. They find that the natural rates in both countries are mainly determined by their own trend in growth of potential output. Grossman et al. (2019) estimate the natural rate of interest for six open economies (Australia, Canada, South Korea, Sweden, Switzerland and the United Kingdom) with a structural open economy model that incorporates exogenous domestic technological progress (trend in growth) and a more flexible representation of preferences for domestic households. Their results show that (i) the natural rates in all six countries have

[2] Loukoianova (2008) finds the puzzle of high interest rates exists in Mongolia by comparing nominal and real interest rates between Mongolia and its peer countries (i.e., Central and Eastern Eurobarometer (CEEB), Commonwealth of Independent States (CIS) and emerging Asian countries).

shifted downwards and exhibited strong co-movements, and (ii) foreign output shocks (spillovers from the rest of the world) are a major factors in the dynamics of the natural rate in these six small open economies. A multi-country structural model for the Euro Area and the United States is developed by Hlédik and Vlček (2018a). In contrast to open versions of the Laubach and Williams (2003) methodology, their semi-structural model is forward-looking and closed by a monetary policy rule, and the natural rate of interest converges to trend in output growth with some persistence. Hlédik and Vlček (2018b) use a single country variant of the model to quantify the natural rate of interest for Czech Republic. In this case, they further modified their model by assuming that the natural rate of interest is linked to trend in output growth adjusted for equilibrium real exchange rate appreciation with some persistence. Different variants of Hlédik and Vlček (2018b) model may include Kirker (2008) and Daníelsson et al. (2016).

The remainder of this paper is structured as follows. Section 2 describes the model applicable to measure the natural rate of interest for commodity exporting economies. Section 3 discusses data, estimation, and the model evaluation. Section 4 discusses the impulse response of the model, dynamics of the estimated natural rate of interest and other unobserved variables, variance and historical decompositions of the natural real rate of interest. Section 5 concludes the paper with implications of the findings.

2. The model

The model used in this paper is a structural New Keynesian model of a small open economy with floating exchange rate. The basic structure of the model mainly builds on Berg et al. (2006) and is based on a standard two country framework where the domestic economy is assumed to be a small open economy and the foreign economy represents the rest of the world. Since it is a log-linearized version of the micro-founded New Keynesian open economy model such as Galí and Monacelli (2005) and Justiniano and Preston (2010), the model is forward looking and closed by a monetary policy rule. It is in contrast to open economy versions built by Laubach and Williams (2003), but in line with the one in Hlédik and Vlček (2018a,b).

The model is further modified in two respect. First, as suggested by Hlédik and Vlček (2018b) for a small open economy with free capital flow such as Mongolia, we assume that the natural rate of interest converges to trend in output growth adjusted for real exchange rate appreciation in equilibrium with some persistence. Second, to reflect the fact that commodity exporting economies are frequently hit by external shocks, we assume that the aggregate demand and the equilibrium exchange rate depend on global commodity price, commodity demand, and FDI. In what follows, we describe the model in detail. The model has six core equations: Natural rate of interest equation, an IS curve, New Keynesian Phillips curve, modified uncovered interest rate parity condition, the equilibrium real exchange rate, and a Taylor-type monetary policy rule.

As suggested by Laubach and Williams (2003), we linked the evolution of the natural real rate of interest with the trend in growth rate of equilibrium output. Since interest rates are defined as per annum in the model, annual trend in growth rate of equilibrium output $(y_t^* - y_{t-4}^*)$ is incorporated in the equation for the natural rate. However, the specification alone is not suitable for a small open economy with free capital flow such as Mongolia. Though the equilibrium output growth rate indicates the capital yield from production, one should consider the fact that real exchange rate appreciation increases the yield on investments made in foreign currency. Hence, in line with Hlédik and Vlček (2018b), the natural real rate of interest is assumed to converge to equilibrium real output growth adjusted for equilibrium real exchange rate appreciation with some persistence:

$$r_t^* = \rho_{r^*} r_{t-1}^* + (1-\rho_{r^*})\left[w_y\left(y_t^* - y_{t-4}^*\right) + \left(1-w_y\right)\left(q_t^* - q_{t-4}^*\right)\right] + \varepsilon_t^{r^*} \tag{1}$$

where ρ_{r^*} is a smoothing parameter in the level of the natural real rate of interest, w_y is the weight on equilibrium real output growth, y_t^* is the equilibrium real output, q_t^* is the equilibrium real exchange rate and $\varepsilon_t^{r^*}$ is a shock to the natural rate of interest. We introduced persistence and a stochastic term ($\varepsilon_t^{r^*}$ shock) in the generalized equation to formally test which specification (equation (1) ($\rho_{r^*} > 0$) versus a random walk ($\rho_{r^*} = 1$)) is supported by the data. Equation (1) links the natural rate with its two most important stationary determinants, $(y_t^* - y_{t-4}^*)$ and $(q_t^* - q_{t-4}^*)$. In this sense, our specification is closer to that of Mésonnier and Renne (2007) and Fries et al. (2018). Since the equilibrium real output growth adjusted for equilibrium real exchange rate appreciation is a component in equation (1), we assign weights on $(y_t^* - y_{t-4}^*)$ and $(q_t^* - q_{t-4}^*)$, which help to estimate impact the determinants on the natural rate, separately. In our specification, the equilibrium real appreciation of the exchange rate taxes the implication of potential GDP growth on the natural rate of interest because of the Penn effect (i.e., whenever a country is growing fast and its per capita GDP converges to that of advanced economies, the growth is accompanied by real appreciation $(q_t^* - q_{t-4}^* < 0)$). Without this adjustment, the natural rate of interest would be unrealistically high due to rapid growth of GDP, especially in emerging and developing countries.

The dynamics of output gap (aggregate demand) are described by a forward-looking IS-equation of the form:

$$\tilde{y}_t = \left(1-\beta_y\right)E_t\tilde{y}_{t+1} + \beta_y\tilde{y}_{t-1} - \beta_r\tilde{r}_{t-1} - \beta_q\tilde{q}_t + \beta_f\tilde{y}_t^f + \beta_{x^{com}}\tilde{x}_t^{com} + \beta_{p^{com}}\tilde{p}_t^{com} + \beta_{fdi}\widetilde{fdi}_t + \varepsilon_t^{\tilde{y}} \tag{2}$$

where current output gap ($\tilde{y}_t$), depends on last period's real activity ($\tilde{y}_{t-1}$), next period's expected real activity ($E_t\tilde{y}_{t+1}$), last period's real interest rate gap ($\tilde{r}_{t-1}$), current period's real exchange rate gap ($\tilde{q}_t$), foreign output gap (foreign demand) ($\tilde{y}_t^f$), gap in demand for commodity export ($\tilde{x}_t^{com}$), commodity price gap ($\tilde{p}_t^{com}$), FDI gap ($\widetilde{fdi}_t$) and aggregate demand shock ($\varepsilon_t^{\tilde{y}}$). Similar specifications of the IS relationship can be found in related papers (i.e., Holston et al., 2017; Hlédik and Vlček, 2018b, Grossman et al., 2019). Deviating from the existing literature, we explicitly incorporate cyclical components of demand for commodity exports, commodity price, and FDI into the IS equation to account for the fact that business cycle fluctuations in Mongolia are largely driven by shocks of FDI, commodity price and commodity demand, found in Gan-Ochir and Davaajargal (2019).

Inflation is modeled using a hybrid New Keynesian Phillips curve in a similar fashion of Andrle et al. (2013) and Berger and Kempa (2014):

$$\pi_t = (1-\alpha_\pi)E_t\pi_{t+1} + \alpha_\pi\pi_{t-1} + \alpha_y\tilde{y}_{t-1} - \alpha_q\tilde{q}_t + \varepsilon_t^\pi \tag{3}$$

where current inflation rate (π_t) depends on last period's inflation (π_{t-1}), next period's expected inflation ($E_t\pi_{t+1}$), last period's real activity ($\tilde{y}_{t-1}$), current period's real exchange rate gap ($\tilde{q}_t$), and cost-push shock (ε_t^π).

Following the approach of Berg et al. (2006) and Benés et al. (2008), the real exchange rate is modeled using a modified Uncovered Interest Rate Parity (UIP) condition given by:

$$q_t = q_{t+1|t}^e + \left(r_t - r_t^f + \rho_t^*\right)\Big/4 + \varepsilon_t^q \tag{4}$$

where q_t is the real exchange rate defined as the price of foreign currency in terms of domestic currency, $q_{t+1|t}^e$ is expected real exchange rate in next period, ρ_t^* is the equilibrium risk premium and ε_t^q is a shock to the risk premium. Since the real interest rate terms are in annual terms, it is divided by 4. Similar to the modification in the UIP condition, the expectation of the real exchange rate ($q_{t+1|t}^e$) is formed as a standardized linear combination of forward-looking rational expectations, E_tq_{t+1}, and adaptive (backward looking) expectations, q_{t-1}:

$$q_{t+1|t}^e = \delta_q E_t q_{t+1} + \left(1-\delta_q\right)q_{t-1} \tag{5}$$

The equilibrium risk premium (ρ_t^*) is defined as:

$$\rho_t^* = 4\left[q_t^* - \delta_q E_t q_{t+1}^* - (1-\delta_z)q_{t-1}^*\right] - r_t^* + r_t^{f,*} \tag{6}$$

where q_t^* is the equilibrium level of the real exchange rate, r_t^* is the natural real rate of interest of the domestic economy, and $r_t^{f,*}$ is the natural real rate of interest in the foreign economy.[3] When expectations are fully rational (i.e., $\delta_q = 1$), equation (4) becomes the standard UIP condition in a gap form.

In line with the literature on macroeconomic gap modeling (i.e., Berg et al., 2006; Andrle et al., 2013), we assume that the equilibrium exchange rate is influenced by the commodity demand, commodity price, and FDI. As Mongolia is a commodity dependent economy, these factors are fundamental determinants of foreign currency supply in the foreign exchange market. Change in equilibrium real exchange rate (Δq_t^*) is assumed to follow the equation below:

$$\begin{aligned}\Delta q_t^* &= \rho_{q^*}\Delta q_{t-1}^* + \\ &+ \left(1-\rho_{q^*}\right)\left[w_{x^{com}}\Delta x_t^{com} + w_{p^{com}}\Delta p_t^{com} + \left(1 - w_{x^{com}} - w_{p^{com}}\right)\Delta fdi_t\right] + \varepsilon_t^{\Delta q^*}\end{aligned} \tag{7}$$

where Δx_t^{com} is change in demand for commodity exports, Δp_t^{com} is change in commodity price, Δfdi_t is change in FDI, and $\varepsilon_t^{\Delta q^*}$ is a shock to change in equilibrium real exchange rate. The presence of the external factors in equation (7) builds a channel in which external shocks affect the natural rate of interest. A transitory shock ($\varepsilon_t^{\Delta q^*}$) is incorporated in line with the suggestion made by Lewis and Vazquez-Grande (2018) that non-growth component of the natural real rate of interest should be modeled as having transitory shocks. However, we formally test whether the specification in equation (7) is supported by the data or not in Section 3.4.

We assume that monetary policy instrument is based on short-term nominal interest rate. The central bank is assumed to conduct monetary policy according to the following forward-looking Taylor-type rule (i.e., to anchor inflation to a stable level):

$$i_t = \gamma_i i_{t-1} + (1-\gamma_i)\left[r_t^* + E_t\pi_{t+1}^* + \gamma_\pi E_t\left(\pi_{t+4}^A - \pi_{t+4}^{A,*}\right) + \gamma_y\tilde{y}_t\right] + \varepsilon_t^i \tag{8}$$

where γ_i is a smoothing parameter in the level of the nominal interest rate (i_t), ε_t^i is monetary policy shock, and π_t^* is the (annualized) inflation target. While it is a quarterly model, interest rates are defined as per annum in the model. In practice, most central banks (including Bank of Mongolia) target annual inflation rate and change the policy interest rate to influence the main macroeconomic variables (e.g., consumer prices and production), ultimately to achieve its inflation target. Hence, in equation (8), we assume that the central bank moves the nominal interest rate in response to deviations in expected annual inflation from its annual target, $E_t(\pi_{t+4}^A - \pi_{t+4}^{A,*})$, and the current output gap ($\tilde{y}_t$). Similar specifications of the rule targeting the annual inflation can also be found in Berg et al. (2006), Andrle et al. (2013) and Cahn et al. (2018).

The remaining equations of the model explicitly define the unobserved variables and their dynamics. The output (y_t), the potential output (y_t^*), domestic ex-ante real interest rate (r_t), the real interest rate gap ($\tilde{r}_t$), the real exchange rate gap ($\tilde{q}_t$), the equilibrium exchange rate (q_t^*), change in demand for commodity export (Δx_t^{com}), change in commodity price (Δp_t^{com}), change in FDI (Δfdi_t), change in

3 In the empirical analysis, we assume $r_t^{f,*} = 0$ for the de-trended data as employed by other studies (i.e., Kirker, 2008 and Daníelsson et al., 2016).

gap of demand for commodity export demand ($\Delta\tilde{x}_t^{com}$), change in commodity price gap ($\Delta\tilde{p}_t^{com}$), change in FDI gap ($\Delta\widetilde{fdi}_t$), annual inflation (π_t^A), annual inflation target ($\pi_t^{A,*}$), and foreign ex-ante real interest rate (r_t^f) are defined as follows:

$$y_t = y_t^* + \tilde{y}_t \tag{9}$$

$$y_t^* = \quad g_t^* + y_{t-1}^* \quad 10)$$

$$r_t = i_t - E_t\pi_{t+1} \tag{11}$$

$$\tilde{r}_t = r_t - r_t^* \tag{12}$$

$$\tilde{q}_t = q_t - q_t^* \tag{13}$$

$$q_t^* = \Delta q_t^* + q_{t-1}^* \tag{14}$$

$$\Delta x_t^{com} = \Delta x_t^{com,*} + \Delta\tilde{x}_t^{com} \tag{15}$$

$$\Delta p_t^{com} = \Delta p_t^{com,*} + \Delta\tilde{p}_t^{com} \tag{16}$$

$$\Delta fdi_t = \Delta fdi_t^* + \Delta\widetilde{fdi}_t \tag{17}$$

$$\Delta\tilde{x}_t^{com} = \tilde{x}_t^{com} - \tilde{x}_{t-1}^{com} \tag{18}$$

$$\Delta\tilde{p}_t^{com} = \tilde{p}_t^{com} - \tilde{p}_{t-1}^{com} \tag{19}$$

$$\Delta\widetilde{fdi}_t = \widetilde{fdi}_t - \widetilde{fdi}_{t-1} \tag{20}$$

$$\pi_t^A = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4 \tag{21}$$

$$\pi_t^{A,*} = \left(\pi_t^* + \pi_{t-1}^* + \pi_{t-2}^* + \pi_{t-3}^*\right)/4 \tag{22}$$

$$r_t^f = i_t^f - E_t\pi_{t+1}^f \tag{23}$$

The equilibrium output growth rate (g_t^*), inflation target (π_t^*), change in equilibrium level of demand for commodity export ($\Delta x_t^{com,*}$), change in equilibrium commodity price ($\Delta p_t^{com,*}$), and change in equilibrium FDI (Δfdi_t^*) are assumed to follow a random walk process:

$$g_t^* = g_{t-1}^* + \varepsilon_t^{g^*} \tag{24}$$

$$\pi_t^* = \pi_{t-1}^* + \varepsilon_t^{\pi^*} \tag{25}$$

$$\Delta x_t^{com,*} = \Delta x_{t-1}^{com,*} + \varepsilon_t^{\Delta x^{com,*}} \tag{26}$$

$$\Delta p_t^{com,*} = \Delta p_{t-1}^{com,*} + \varepsilon_t^{\Delta p^{com,*}} \tag{27}$$

$$\Delta fdi_t^* = \Delta fdi_{t-1}^* + \varepsilon_t^{\Delta fdi^*} \tag{28}$$

where $\varepsilon_t^{\Delta g^*}$, $\varepsilon_t^{\pi^*}$, $\varepsilon_t^{\Delta x^{com,*}}$, $\varepsilon_t^{\Delta p^{com,*}}$ and $\varepsilon_t^{\Delta fdi^*}$ are shocks to the equilibrium output growth, inflation target, change in equilibrium level of demand for commodity export, change in equilibrium commodity price and change in equilibrium FDI, respectively.

The foreign output gap ($\tilde{y}_t^f$), foreign inflation (π_t^f) and foreign nominal interest rate (i_t^f), gap in demand for commodity export ($\tilde{x}_t^{com}$), commodity price gap ($\tilde{p}_t^{com}$) and FDI gap ($\widetilde{fdi}_t$) are assumed to follow AR(1) process:

$$\tilde{y}_t^f = \rho_y^f \tilde{y}_{t-1}^f + \varepsilon_t^{y,f} \tag{29}$$

$$\pi_t^f = \rho_\pi^f \pi_{t-1}^f + \varepsilon_t^{\pi,f} \tag{30}$$

$$i_t^f = \rho_i^f i_{t-1}^f + \varepsilon_t^{i,f} \tag{31}$$

$$\tilde{x}_t^{com} = \rho_{x^{com}} \tilde{x}_{t-1}^{com} + \varepsilon_t^{\tilde{x}^{com}} \tag{32}$$

$$\tilde{p}_t^{com} = \rho_{p^{com}} \tilde{p}_{t-1}^{com} + \varepsilon_t^{\tilde{p}^{com}} \tag{33}$$

$$\widetilde{fdi}_t = \rho_{fdi} \widetilde{fdi}_{t-1} + \varepsilon_t^{\widetilde{fdi}} \tag{34}$$

Since the foreign variables consist of series from different countries (i.e., the Mongolian business cycle is more influenced by Chinese real GDP, however the real exchange rate are largely affected by the US interest rate and inflation), we specify AR(1) process for each series rather than using a VAR system. In the specifications of demand for commodity export and commodity price, we consider the empirical finding documented by Chen et al. (2008): commodity markets are largely independent of the developments in an exporting country and can be treated as exogenous for these countries. Following Gan-Ochir and Undral (2018), FDI is also modeled as an exogenous process, reflecting the fact that the directions of FDI flows are uncertain. It is usually the case for resource rich countries where FDI mainly flows to the mining sector. For example, Mongolia faced sudden flood of FDI during 2010–2013 encouraged by the development of the Oyu Tolgoi (OT) copper and gold deposit, which is the largest foreign investment project ever in Mongolia and attracted more than $6 billion (50 per cent of GDP) in FDI for its first phase. However, sudden stop of FDI occurred in 2014 and remained subdued until 2017 because of sharp decline in commodity prices, completion of the first phase of the OT project, and political risk and uncertainties surrounding the mining sector.

The model equations (1)–(34) can be cast into a linear Gaussian state-space model of the following general form:

$$Y_t = HS_t + AX_t + u_t,\ u_t \sim N(0, R) \tag{35}$$

$$S_t = FS_{t-1} + v_t,\ v_t \sim N(0, Q) \tag{36}$$

where Y_t is a $p \times 1$ vector of p observed endogenous variables modeled in the observation equation (35), X_t is a vector of exogenous and pre-determined variables and S_t is a $m \times 1$ vector of m unobserved states modeled in the state equation (36). The vectors u_t and v_t are assumed to be Gaussian and mutually uncorrelated with mean zero and covariance matrices R and Q, respectively. Since it is a linear rational expectations (RE) model with higher order forward looking ($E_t \pi_{t+4}^A$ and $E_t \pi_{t+4}^{A,*}$) and lagged (y_{t-4}^* and q_{t-4}^*) variables, we use Dynare 4.5.4 to solve exogenous vector (X_t), the state vector (S_t), the vectors of stochastic disturbances (u_t, v_t) and coefficient matrices (H, A, F, R, Q). Once the RE model is solved as the contemporaneous endogenous variables are determined by lagged state variables and contemporaneous disturbances, the expectations of endogenous variables are obtained from the model. As it is a Gaussian state-space model, likelihood of the model can be evaluated with the Kalman filter.

Using the Kalman smoother, we are able to back-out estimates of the series for unobservable natural rate of interest (r_t^*), inflation target (π_t^*), growth rate of equilibrium output (g_t^*) and equilibrium real exchange rate (q_t^*), once the model is estimated.

3. Data and estimation

3.1. Data

The model is estimated using quarterly data from 2000Q1 to 2018Q4, which is the longest available time series as the quarterly data for Mongolian GDP is only available since 2000Q1. For the domestic economy (Mongolia), the following seven variables are observed. GDP growth is measured as the log-difference of seasonally-adjusted real GDP in percent ($dlGDP_t$). The nominal interest rate is defined as the annual weighted average of central bank bill (CBB) rate in percent ($CBBRATE_t$).[4] Inflation is an annualized measure derived as the log-difference of the seasonally adjusted consumer price index (CPI) in percent ($4 \times dlCPI_t$). Change in log real exchange rate ($dlRER_t$) is derived as the log-difference of RER_t, where $RER_t = 100 \times \log(e_t \times CPI_t / CPI_t^f)$, e_t is the nominal exchange rate (USD/MNT), and inflation rates for Mongolia and the United States are measured by their respective CPIs, CPI_t and CPI_t^f. The change in log real commodity export is measured as the log-difference of seasonally-adjusted real commodity export[5] ($dlREXPCOM_t$) and the change in log of net inward FDI is derived as the log-difference of FDI ($dlFDI_t$) (all scaled by 100). Real GDP (at 2010 constant prices) and CPI are collected from the website of the National Statistical Office of Mongolia (www.1212.mn), and the CBB rate, nominal exchange rate, nominal commodity export (in USD) and net inward FDI (in USD) are obtained from statistical bulletins of the Bank of Mongolia.

Four foreign variables are used in the estimation. Mongolia has a close trade link with China, while is influenced by the US financial variables. For instance, exports to China accounts for 90 percent of total exports, suggesting that external demand is more influenced by Chinese economy. However, 80 percent of export revenues and 65 percent of import payments are paid by USD.[6] Share of USD deposits

[4] The weighted average CBB rate has been closer to the policy rate since July 2007 when the policy rate is introduced as the 7-day CBB rate.
[5] The real commodity export is measured as the nominal commodity export to commodity price ratio.
[6] Share of Chinese Yuan (CNY) is 18 percent for export revenues and 15 percent for import payments.

in the banking sector accounts for 27 percent of total deposits, while share of CNY deposits is less than 2 percent. Therefore, exchange rate of Mongolian tögrög (MNT) against USD and US interest rate are key variables, affecting domestic pricing of goods and financial products. Due to abovementioned stylized facts, China's output is used as a proxy for the foreign output, while US interest rate and US inflation are used to proxy the foreign interest rate and foreign inflation, respectively. As the foreign variables are modeled as exogenous AR(1) process in the model, using observable foreign variables from different countries does not violate the theoretical setting of the model. The foreign output gap is calculated using an HP filter on the log of seasonally adjusted real GDP of China ($OUTGAP_t^f$). Foreign nominal interest rate is defined as effective Federal Funds rate per annum (quarterly average) ($FEDFUNDS_t^f$). Foreign annualized inflation ($4\times dlCPI_t^f$) is measured as the log-difference of the seasonally adjusted US CPI. The change in log commodity price is measured as the log-difference of commodity price index (*dlPCOM*) (all scaled by 100). All observed foreign variables are in percent. China's real GDP is calculated[7] using real GDP YoY growth, directly observed from the Bloomberg database, while effective Federal Funds rate (not seasonally adjusted, in percent) and US CPI (all items for the US, index 2015 = 100, seasonally adjusted) are collected from FRED economic database, Federal Reserve Bank of St. Louis. The commodity price index (non-rural commodity prices in USD, GRCPNRUSD) is obtained from the commodity price statistics (I2 table) of Reserve Bank of Australia.

The corresponding measurement equation is:

$$Y_t = \begin{bmatrix} dlGDP_t \\ 4\times dlCPI_t \\ CBBRATE_t \\ dlRER_t \\ dlREXPCOM_t \\ dlFDI_t \\ dlPCOM_t \\ OUTGAP_t^f \\ 4\times dlCPI_t^f \\ FEDFUNDS_t^f \end{bmatrix} = \begin{bmatrix} y_t - y_{t-1} \\ \pi_t \\ i_t \\ q_t - q_{t-1} \\ \Delta x_t^{com} \\ \Delta fdi_t \\ \Delta p_t^{com} \\ \tilde{y}_t^f \\ \pi_t^f \\ i_t^f \end{bmatrix} \tag{37}$$

where *l* and *dl* stand for 100 times log and log difference, respectively.

3.2. A Bayesian inference and priors of the parameters

We estimate non-calibrated parameters (θ) of the model using Bayesian estimation techniques.[8] In the Bayesian framework, a prior distribution on parameters $p(\theta)$ is updated by sample information contained in the likelihood function $L(Y^T|\theta)$ to form a posterior distribution $p(\theta|Y^T)$.

$$p(\theta|Y^T) \propto L(Y^T|\theta)p(\theta) \tag{38}$$

The prior is based on 'non-sample' information, so that the Bayesian methods provide ideal framework for combining different sources of information (Del Negro and Schorfheide, 2011). Since the mapping from the DSGE model to its $L(Y^T|\theta)$ is nonlinear in θ, construction of the posterior distribution is too complicated to evaluate analytically. Hence, simulation techniques such as Markov chain Monte Carlo (MCMC) methods with the likelihood obtained at each draw through the Kalman filter are used to obtain draws from the posterior distribution shown in equation (38). When estimating a structural model, the choice of MCMC procedure is usually the Random Walk Metropolis (RWM) algorithm, which belongs to a more general class of Metropolis-Hastings algorithms. A detailed discussion of numerical techniques such as the RWM and Kalman filter algorithms is provided in An and Schorfheide (2007), Guerrón-Quintana and Nason (2012), and Del Negro and Schorfheide (2011). The Bayesian framework naturally focuses on the evaluation of relative fitness of the model. Bayes factors or posterior odds ratios[9] are used to measure the relative merits amongst a number

[7] We collect China's real GDP data for 4 quarters of 2018 from National Bureau of Statistics of China, then real GDP for the period of 2000Q1-2017Q4 are calculated using the Chinese YoY growth, observed from the Bloomberg database.

[8] Bayesian methods help estimate models with cross-equation restrictions by dealing with misspecification and identification problems, well. In the presence of those problems, advantages of the approach over alternatives are discussed by Canova (2007) and An and Schorfheide (2007).

[9] If there are M competing models, and one does not have strong view on which model is the best one (i.e., hence chooses equal prior weight for each model, $1/M$), the posterior odds ratio is equal to the Bayes factor.

of competing models. The Bayes factor of model $\mathscr{M}_j$ versus model $\mathscr{M}_s$ is given by

$$\mathscr{BF}_{j,s|Y^T} = p\left(Y^T\middle|\ \mathscr{M}_j\right)/p\left(Y^T\middle|\ \mathscr{M}_s\right) \tag{39}$$

which summarizes the sample evidence in favor of $\mathscr{M}_j$ over $\mathscr{M}_s$. The terms $p(Y^T|\ \mathscr{M}_j)$ and $p(Y^T|\ \mathscr{M}_s)$ are the marginal likelihoods of $\mathscr{M}_j$ and $\mathscr{M}_s$, respectively. The marginal likelihood for a given model $\mathscr{M}_i$ is calculated as $p(Y^T|\ \mathscr{M}_i) = \int_\theta L(Y^T|\theta,\mathscr{M}_i)p_i(\theta|\mathscr{M}_i)d\theta$, where $L(Y|\theta,\mathscr{M}_i)$ is the likelihood function for the data Y^T conditional on the parameter vector and on the model. The marginal likelihood measure automatically penalizes models with additional parameters and increasing degrees of complexity.

Table 1
Prior densities and posterior estimates.

Parameters		Prior Distribution				Posterior Distribution	
		Density	Mean	Sd.	[5,95] prob.	Mean	[5,95] prob.
Natural interest rate (NIR)							
ρ_{r^*}	NIR: smoothing	B	0.7	0.2	[0.32, 0.96]	0.96	[0.92, 0.99]
w_y	NIR: weight on potential growth	B	0.6	0.2	[0.25, 0.90]	0.64	[0.36, 0.94]
IS curve							
β_y	IS: weight on lag	B	0.5	0.2	[0.17, 0.83]	0.17	[0.05,0.28]
β_r	IS: effect of real int.rate gap	G	0.1	0.05	[0.03, 0.19]	0.11	[0.04, 0.17]
β_q	IS: effect of real exch rate gap	G	0.01	0.005	[0.003, 0.02]	0.013	[0.003, 0.02]
β_f	IS: effect of foreign output gap	G	0.05	0.02	[0.02, 0.09]	0.048	[0.02, 0.08]
$\beta_{x^{com}}$	IS: effect of commodity export gap	G	0.05	0.02	[0.02, 0.09]	0.017	[0.01, 0.03]
$\beta_{p^{com}}$	IS: effect of commodity price gap	G	0.05	0.02	[0.02, 0.09]	0.048	[0.02, 0.08]
β_{fdi}	IS: effect of FDI gap	G	0.05	0.02	[0.02, 0.09]	0.008	[0.003, 0.01]
Phillips curve							
α_π	PC: weight on lag	B	0.5	0.2	[0.17, 0.83]	0.11	[0.04, 0.18]
α_y	PC: effect of output gap	G	0.1	0.05	[0.03, 0.19]	0.08	[0.02, 0.13]
α_q	PC: effect of real exch.rate gap	G	0.075	0.05	[0.02, 0.17]	0.02	[0.001, 0.04]
UIP condition and Equilibrium exchange rate (EER)							
δ_q	UIP: forward looking weight	B	0.5	0.2	[0.17, 0.83]	0.78	[0.16, 0.99]
ρ_{q^*}	EER: smoothing	B	0.5	0.2	[0.17, 0.83]	0.43	[0.19, 0.66]
$w_{x^{com}}$	EER: weight on commodity export demand	B	0.33	0.1	[0.17, 0.50]	0.30	[0.17, 0.45]
$w_{p^{com}}$	weight on commodity price	B	0.33	0.1	[0.17, 0.50]	0.30	[0.14, 0.46]
γ_i	MP: smoothing	G	0.7	0.15	[0.47, 0.96]	0.71	[0.60, 0.80]
γ_π	MP: responsiveness to inflation	G	2.0	0.5	[1.25, 2.89]	2.17	[1.37, 2.92]
γ_y	MP: responsiveness to output gap	G	1.0	0.3	[0.56, 1.54]	0.85	[0.46, 1.26]
Foreign economy and external variables							
ρ_y^f	Foreign output gap, AR(1)	B	0.5	0.2	[0.17, 0.83]	0.56	[0.43, 0.70]
ρ_π^f	Foreign inflation, AR(1)	B	0.5	0.2	[0.17, 0.83]	0.48	[0.36, 0.61]
ρ_i^f	Foreign interest rate, AR(1)	B	0.5	0.2	[0.17, 0.83]	0.91	[0.88, 0.94]
$\rho_{x^{com}}$	Comm export demand, AR(1)	B	0.5	0.2	[0.17, 0.83]	0.74	[0.63, 0.86]
$\rho_{p^{com}}$	Comm price, AR(1)	B	0.5	0.2	[0.17, 0.83]	0.69	[0.51, 0.89]
ρ_{fdi}	FDI, AR(1)	B	0.5	0.2	[0.17, 0.83]	0.74	[0.65, 0.83]
Standard deviation of shocks							
$\sigma_{\tilde{y}}$	Sd domestic output	IG	2.0	Inf	[0.42, 5.63]	2.26	[1.87, 2.65]
σ_π	Sd domestic inflation	IG	4.0	Inf	[0.84, 11.3]	7.91	[6.64, 9.22]
σ_i	Sd domestic interest	IG	2.0	Inf	[0.42, 5.63]	2.77	[2.29, 3.21]
σ_{g^*}	Sd potential growth rate	IG	0.25	Inf	[0.05, 0.70]	0.32	[0.16, 0.47]
σ_{π^*}	Sd inflation target	IG	0.25	Inf	[0.05, 0.70]	0.23	[0.06, 0.42]
σ_{r^*}	Sd natural interest rate	IG	0.25	Inf	[0.05, 0.70]	0.61	[0.05, 1.83]
σ_q	Sd real exch rate	IG	2.0	Inf	[0.42, 5.63]	1.44	[0.59, 2.23]
σ_{q^*}	Sd equil real exch rate	IG	2.0	Inf	[0.42, 5.63]	3.89	[2.75, 5.15]
$\sigma_{x^{com}}$	Sd commodity export demand	IG	10.0	Inf	[2.11, 28.1]	15.74	[13.5, 18.1]
$\sigma_{p^{com}}$	Sd commodity price	IG	5.0	Inf	[1.05, 14.1]	4.91	[2.93, 6.77]
σ_{fdi}	Sd FDI	IG	20.0	Inf	[4.22, 56.3]	29.67	[25.1, 33.7]
$\sigma_{\Delta x^{com,*}}$	Sd equil commodity export demand	IG	1.0	Inf	[0.21, 2.81]	3.89	[1.67, 6.15]
$\sigma_{\Delta p^{com,*}}$	Sd equi commodity price	IG	0.5	Inf	[0.11, 1.41]	0.56	[0.12, 1.28]
$\sigma_{\Delta fdi^*}$	Sd equil FDI	IG	0.5	Inf	[0.11, 1.41]	0.33	[0.12, 0.56]
σ_y^f	Sd foreign output	IG	0.5	Inf	[0.11, 1.41]	0.57	[0.48, 0.67]
σ_π^f	Sd foreign inflation	IG	2	Inf	[0.42, 5.63]	2.49	[2.14, 2.82]
σ_i^f	Sd foreign interest rate	IG	0.25	Inf	[0.05, 0.70]	0.41	[0.36, 0.47]

Notes: G: Gamma distribution, B: Beta distribution, IG: Inverse Gamma distribution and 'Inf' represents infinitive. Figures in brackets indicate 90 percent posterior probability intervals.

We estimate the mode of the posterior distribution by maximizing the log posterior function shown in equation (38). In the second step, the RWM algorithm is used to get the posterior distribution and to evaluate the marginal likelihood of the model. All numerical estimations, evaluations and simulations in this paper are done using Dynare. Christopher Sims's 'csminwel' optimization routine is used to obtain the posterior mode and to compute the Hessian matrix at the mode. To test the presence of the identification problem more than 20 optimization runs are launched. Different optimization routine always converges to the same mode value. Since a unique mode for the model is found, the Hessian from the optimization routine is used as a proposal density, properly scaled ($c = 0.32$) to attain an acceptance rate of 27 percent. For the RWM results, two independent chains are generated with 100,000 draws each, of which 40,000 are used as an initial burn-in phase. Convergence of the chains is monitored using both the univariate and the multivariate convergence diagnostics variants of Brooks and Gelman (1998).

To estimate the parameters using Bayesian estimation, prior distributions must be specified for each parameter in the model. The priors are described in the first panel of Table 1. The priors are fairly uncontroversial with previous studies using Bayesian inference. However, the choice of prior is influenced by a range of previous models of the Mongolian economy. For instance, we try to find reasonable prior values for weakly identified parameters using other studies such as Gan-Ochir and Dulamzaya (2014), Gan-Ochir and Undral (2018), which estimate similar models for Mongolia and Bumchimeg et al. (2013), which is an earlier version of the model currently used at the Bank of Mongolia for monetary policy analysis and forecasting.

For natural real rate of interest dynamics, the prior mean on degree of interest-smoothing (ρ_{r^*}) and the sensitivity on the equilibrium output growth (w_y) are set at 0.7 and 0.6, respectively. The values are almost same as the estimated value in Hlédik and Vlček (2018b).

We choose relatively diffuse priors on the lagged values of output in IS curve (β_y) and inflation in Phillips curve (α_π). In line with Gan-Ochir and Dulamzaya (2014), the parameters have beta distributions with mean of 0.5 and standard deviation of 0.2. We choose a prior mean of 0.1 (with standard deviation of 0.05) for the sensitivity of the domestic output gap to the real interest rate gap (β_r), and a prior mean of 0.01 for the sensitivity of the domestic output gap to the real exchange rate gap (β_q). These are same priors as used by Kirker (2008) and quite close to the values calibrated in Bumchimeg et al. (2013). The priors for the sensitivity of the domestic economy to foreign demand (β_f), gap in demand for commodity export ($\beta_{x^{com}}$), commodity price gap ($\beta_{p^{com}}$) and FDI gap (β_{fdi}) are chosen as gamma distribution with mean of 0.05. It is in line with the following fact: (i) although China is a large export market for Mongolia, it is not dominant enough that minor changes in the foreign demand would create significant changes in demand pressures in Mongolia, and (ii) the sensitivity parameters will be low since volatilities in the commodity price, demand for commodity export, and FDI are greater than fluctuations in the domestic output gap (Gan-Ochir and Undral, 2018).

In the Phillips curve, a gamma distribution with mean of 0.1 (and standard deviation of 0.05) is assigned to the prior for the sensitivity of output gap on inflation (α_y). It is in line with the literature that this parameter value ranges in size from 0.0011 (Buncic and Melecky, 2008 for Australia) to 0.22 (Harjes and Ricci, 2008 for South Africa) for small open economies. The prior for the sensitivity of domestic inflation to an appreciation in the real exchange rate (α_q) is chosen as a gamma distribution with mean of 0.075 (and standard deviation of 0.05), a relatively loose prior.

We set a beta distribution with mean of 0.5 for the prior on the sensitivity of expected real exchange rate to next period's rationally expected value (δ_z), since the parameter has to take a positive value. The prior of $\delta_z = 0.5$ is in line with the empirical fact that the standard UIP condition does not hold and exchange rate persistence exists in Mongolia (i.e., Gan-Ochir and Dulamzaya, 2014; Gan-Ochir and Undral, 2018). We also set a diffuse prior on degree of real exchange rate-smoothing (ρ_{q^*}) as beta distribution with mean of 0.5.

As three external factors are included in the equilibrium real exchange rate equations, we set the priors on $w_{p^{com}}$ and $w_{x^{com}}$ as beta distributions with mean of 0.33.

Regarding to the monetary policy rule, the prior on degree of interest-smoothing (γ_i) is set as the gamma distribution with mean of 0.7 and standard deviation of 0.15. This prior is in line with the estimated values in Gan-Ochir and Undral (2018). In line with Berg et al. (2006), we set a diffuse prior on the monetary policy responsiveness to expected inflation deviations from the target (γ_π) as a gamma distribution with mean of 2.0 and standard deviation of 0.5, implying that Bank of Mongolia responds rather aggressively towards deviations in inflation from its target.

In line with models previously estimated for Mongolia, we set the prior mean on the sensitivity to the output gap (γ_y) to 1, reflecting the fact that the Bank of Mongolia also considers economic activity. As a common choice, beta distributions with mean of 0.5 are set for the autoregressive parameters in AR(1) processes, while inverse gamma distribution is selected for all standard deviations of shocks, and prior variances are chosen as diffuse for shocks. As observed from the result estimated by Gan-Ochir and Undral (2018), relatively high values of prior means are set for the shocks of demand for commodity export and FDI.

3.3. Posterior estimates of the parameters

This paper is the first attempt to estimate the reduced-from gap model using Bayesian methods in the case of Mongolia. Previous papers (i.e., Dutu, 2012, Gan-Ochir and Dulamzaya, 2014 and Gan-Ochir and Undral, 2018) have estimated log-linearized DSGE models using Bayesian methods, while Bumchimeg et al. (2013) calibrated their gap model based on the OLS estimations of some equations.

The last two columns in Table 1 report the posterior estimates (i.e., posterior mean and 90 percent probability interval of the posterior distribution) of the model parameters. The data significant information regarding the values of the model parameters, as the posteriors shift significantly from the priors. This implies that the estimated model reflects certain features of the Mongolian economy. Considering the focus of this paper, only selected parameters are explained here.

The results show a relatively high persistence in the natural rate of interest ($\rho_{r^*} = 0.96$), and as suggested by the theory (i.e., Euler equation) annual growth of potential output significantly affects the natural rate of interest ($w_y = 0.64$). However, persistence in the IS-

Table 2
Relative fit of alternative specifications.

Models ($\mathscr{M}$)	Log marginal data densities ($ln\,L(Y^T\|\mathscr{M}_i)$)	Bayes factor ($\mathscr{BF}$)
$\mathscr{M}_0$: Baseline model ($\rho_{r^*}, \rho_{q^*} > 0$)	−2376.33	$\mathscr{BF}_{0,0\|Y} = 1$
$\mathscr{M}_1$: Model with ($\rho_{r^*} = 1, \rho_{q^*} > 0$)	−2379.93	$\mathscr{BF}_{0,1\|Y} = 36.6$
$\mathscr{M}_2$: Model with ($\rho_{r^*} > 0, \rho_{q^*} = 1$)	−2385.16	$\mathscr{BF}_{0,2\|Y} = 6836.3$

curve is relatively low, where the weight on lagged output gap is estimated at $\beta_y = 0.17$. The low persistence is also estimated in Gan-Ochir and Dulamzaya (2014). The estimation also shows that the domestic output gap is relatively insensitive to the real exchange rate and foreign demand conditions, similar to prior beliefs. However, the posterior of the parameter on real interest rate gap is higher than the prior, implying the monetary policy is effective in managing the domestic economic activity. As shown by previous studies (i.e., Gan-Ochir and Undral, 2018; Gan-Ochir and Davaajargal, 2019), the demand for commodity export, commodity price and FDI significantly affect the domestic business cycle.

The domestic Phillips curve is predominantly forward-looking ($\alpha_\pi = 0.11$). The Phillips curve also shows low sensitivity to domestic output gap and the real exchange rate gap. The result is in line with the fact that inflation fluctuations in Mongolia were mainly driven by supply factors such as increases in fuel and meat prices. However, the estimated parameter governing the effect of the domestic economic activity on inflation is significant in the sense that the 90 percent interval does not include zero.

The expectation of the real exchange rate is predominantly forward-looking ($\delta_q = 0.78$), and a moderate level of persistence is observed in the equilibrium real exchange rate ($\rho_{q^*} = 0.43$). The result also shows that demand for commodity export, commodity price and FDI significantly affect the equilibrium real exchange rate.

The Taylor-type rule for monetary policy demonstrates a high degree of persistence ($\gamma_i = 0.71$), suggesting the Bank of Mongolia seeks to smooth changes in the interest rate over time. The estimation results also show that monetary policy is aggressive towards deviations in expected inflation from its target ($\gamma_\pi = 2.17$) and less aggressive towards output deviations ($\gamma_y = 0.85$). The estimated parameters are in line with the existing literature on monetary policy rules in Mongolia.

The estimation also suggests that the effective Federal Fund rate is more persistent compared to Chinese output and the US inflation. The data contains significant information about the parameters of the shock process. In particular, standard deviations in shocks of domestic inflation, commodity export, commodity price and FDI are estimated at relatively high values depending on the fluctuations in the observed variables.

3.4. *The model evaluation*

The Bayes factor is employed to measure the relative merits of the alternative models. Table 2 reports the log marginal data densities of models with different natural rate of interest and equilibrium real exchange rate equations, along with the corresponding Bayes factors, calculated by considering the estimated model (the baseline model) as the null hypothesis. In the baseline model ($\mathscr{M}_0$), shocks to natural real rate of interest ($\varepsilon_t^{r^*}$) and the equilibrium real exchange rate ($\varepsilon_t^{\Delta q^*}$) are transitory shocks as equations (1) and (7) are not random walk processes.

The main interest here is to assess whether (i) the assumption that the natural real rate of interest converges to equilibrium real output growth adjusted for equilibrium real exchange rate appreciation, and (ii) the assumption that the equilibrium real exchange rate is affected by demand for commodity exports, commodity price and FDI. We needed to assess whether these assumptions were supported by the data. Those two assumptions are novel features of our model tailored for Mongolia, a resource-rich developing economy.

Notes: The table reports Bayes factors by comparing the model $\mathscr{M}_0$ to $\mathscr{M}_1$ (or $\mathscr{M}_2$). The log marginal data densities reported here is computed from the posterior draws using the Laplace approximation.

To examine whether the equilibrium real output growth adjusted for equilibrium real exchange rate appreciation has implications on the natural real rate of interest, we compare the baseline model ($\mathscr{M}_0$) with $\mathscr{M}_1$ model where the natural real rate of interest follows a random walk process ($\rho_{r^*} = 1$), similar to the one employed by Kirker (2008) and Us (2018). The marginal data densities of $\mathscr{M}_0$ is larger than the densities of $\mathscr{M}_1$ by 3.6 on a log-scale that translates into a Bayes factor of $\mathscr{BF}_{0,1|Y} = 36.6$. According to Kass and Raftery (1995),[10] the Bayes factor of this size offers 'strong' evidence in favor of $\mathscr{M}_0$ ($\rho_{r^*} > 0$). Considering the Bayes factor as evaluation criterion, the data (Y^T) strongly supports the hypothesis that the natural real rate of interest is determined by the equilibrium real output growth adjusted for equilibrium real exchange rate appreciation. In the following alternative model ($\mathscr{M}_2$), we assume that the equilibrium real exchange rate is a random walk ($\rho_{q^*} = 1$). In such case, the equilibrium exchange rate shock ($\varepsilon_t^{\Delta q^*}$) is estimated as a permanent shock. When comparing the baseline model ($\mathscr{M}_0$) to $\mathscr{M}_2$ model, there is 'very strong' evidence in favor of $\mathscr{M}_0$ model, supporting the assumption that the equilibrium real exchange rate is linked to external factors. The formal test results also show that the natural real rate of interest and the equilibrium real exchange rate are affected by transitory shocks (i.e., $\varepsilon_t^{r^*}$ and $\varepsilon_t^{\Delta q^*}$). Our finding supports the evidence presented by Lewis and Vazquez-Grande (2018) - the natural real rate of interest is affected by both permanent and transitory

[10] According to scale of Kass and Raftery (1995), a Bayes factor between 1 and 3 is 'not worth more than a bare mention', between 3 and 20 suggests a 'positive' evidence, between 20 and 150 suggests a 'strong' evidence, and larger than 150 'very strong' evidence in favor of one of the two models.

shocks. Their results suggest that non-growth component of the natural real rate of interest is subject to persistent, but transitory shocks, which stands in contrast to earlier findings.

4. Empirical results

4.1. Impulse responses

This section aims to check whether the estimated model is able to replicate the stylized facts of the impulse responses, to make sure that the shocks are properly identified. We focus on effects of selected domestic and external shocks on the observed domestic variables. Fig. 1 reports impulse responses of 1 percentage point shock to domestic output gap. The solid and dashed lines respectively show the posterior mean response and the responses of the 90 percent confidence interval.

In response to the domestic output shock, GDP growth immediately increases, and annual interest rate and quarterly inflation rises by 0.25 and 0.06 percentage points, respectively. Because of higher inflation and interest rate, real exchange rate appreciates and they pull down the GDP growth in second quarter. Interest rate exhibits more persistence compared to other variables. The result that a demand shock raises inflation and interest rate in Mongolia is also evident from previous studies (i.e., Gan-Ochir and Davaajargal, 2019).

Fig. 2 shows impulse responses to 1 percentage point increase in the (quarterly) domestic inflation. In response to the shock, domestic interest rate increases, but its magnitude is small compared to the change in inflation, and the decrease in real interest rate pushes GDP growth up in the impact period. The real exchange rate initially depreciates, however the effects on these variables are negligible.

Fig. 3 displays impulse responses to a 100 basis points increase in the (annual) domestic short-term interest rate. The shock has the standard features of an aggregate demand shock. The policy rate shock immediately leads to real appreciation of exchange rate. The rise in interest rate and the real exchange appreciation lower the GDP growth by 0.35 percentage points. As a consequence, inflation decreases by 0.2 percentage points. The effect of monetary policy shock on output and inflation disappears after 10 and 12 quarters, respectively. The findings are in line with previous studies for Mongolia (i.e., Gan-Ochir and Dulamzaya, 2014; Gan-Ochir and Undral, 2018).

Fig. 4 reports impulse responses to a positive shock in natural real rate of interest (i.e., 100 basis points), which is modeled as a transitory shock. As found in Section 3.4, the shock has characteristics of a transitory shock. The shock immediately depreciates the real exchange rate and increases the domestic interest rate. They have opposite effects on GDP growth and the dominant effect of real exchange depreciation initially raises the GDP growth by 0.2 percentage points. As domestic interest rate increases, GDP growth declines over time and the impact of interest rate is more persistent. There is no significant effect on inflation.

Fig. 5 presents impulse responses to 1 percentage point increase in potential output growth. The shock has persistent effects on the domestic economy. For instance, the higher GDP growth, driven by the positive shock, leads to higher inflation and interest rate, but initially depreciates the real exchange rate. The impact on output growth and inflation are noticeably persistent, and interest rate rises over time.

Fig. 6 shows impulse responses to 1 percent increase in equilibrium commodity price. The shock has temporary effect on quarterly GDP growth and persistent effects on the domestic interest rate and inflation. In response to the shocks, real exchange rate appreciates significantly, GDP growth increases by 0.06 percentage points and inflation increases by 0.18 percentage points. The impact on inflation is persistent and real exchange rate appreciates over time. Impulse responses to positive shocks in equilibrium export demand and equilibrium FDI are qualitatively very similar to the responses shown in Fig. 6. The shape of responses and the results highlighting the importance of external shocks in the Mongolian economy are in line with existing literature (i.e., Gan-Ochir and Undral, 2018; Gan-Ochir and Davaajargal, 2019).

Fig. 7 displays impulse responses to 1 percent appreciation in equilibrium real exchange rate, which is modeled as a transitory shock. In response to the shock, domestic interest rate rises and effects on inflation and GDP growth are negligible.

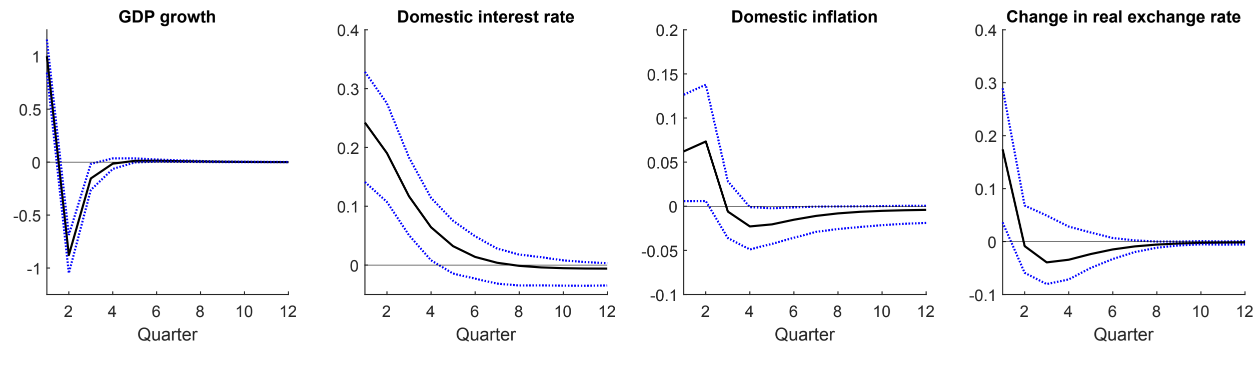

Fig. 1. Impulse response functions to a domestic output gap shock ($\varepsilon_t^{\tilde{y}}$).

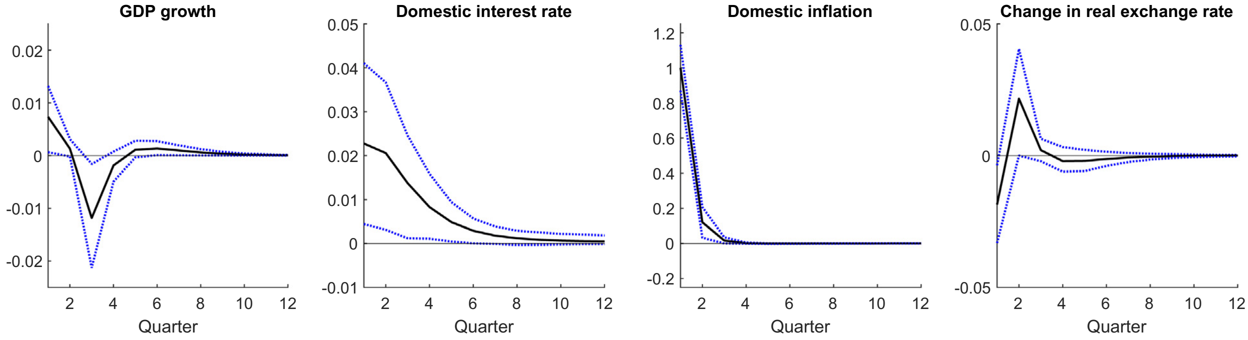

Fig. 2. Impulse response functions to a domestic inflation shock (ε_t^{π}).

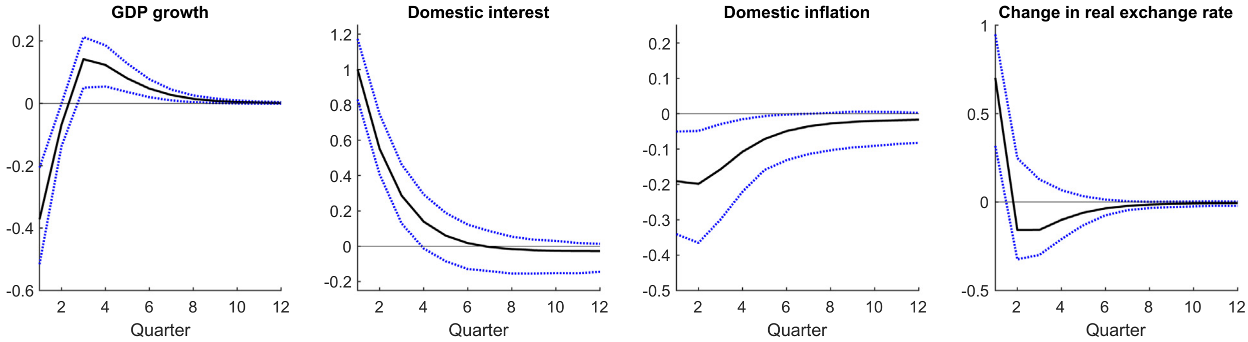

Fig. 3. Impulse response functions to a domestic interest rate shock (ε_t^{i}).

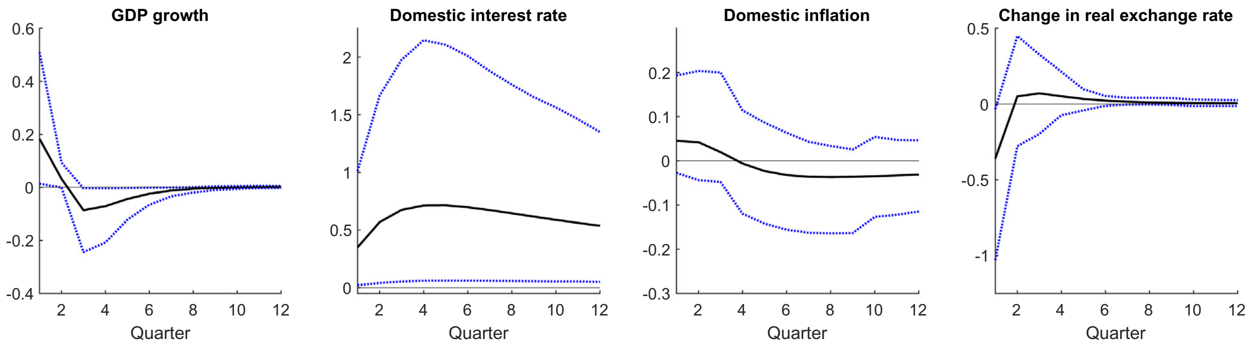

Fig. 4. Impulse response functions to shock in natural real rate of interest ($\varepsilon_t^{r^*}$).

The impulse responses show that the shocks are appropriately identified and the relationship among variables are in line with the open economy macroeconomics. Therefore, in the following section, we use the estimated model to measure the unobservable variables.

4.2. Model estimated natural rate of interest and other unobserved variables

Dynamics of the unobservable variables within the estimated model are extracted using the Kalman smoother. In particular, we find the time-varying estimates of natural real rate of interest, output gap, inflation target, potential output growth, and equilibrium real exchange rate. The results are plotted in Figs. 8–12.

From Figs. 8 and 9, it is clear that the natural real rate of interest for the Mongolian economy is procyclical. The estimated natural real rate of interest in Fig. 8 shows that between 2000 and 2004 the natural real rate of interest was relatively stable around 3 percent. However, starting 2004 the natural real rate was on a downward trend, reaching almost 1.4 percent in 2006 before starting to rise. After 2006, the natural real rate of interest was on an upward trend and reached 5 percent (at the posterior mean) in 2008. Then declined to 3.4 percent (at the posterior mean) in 2009 when the Mongolian economy is hit by the GFC.

Starting from 2010, the natural real rate of interest started to increase and reached 5.9 percent in the end of 2012. It was kept at such

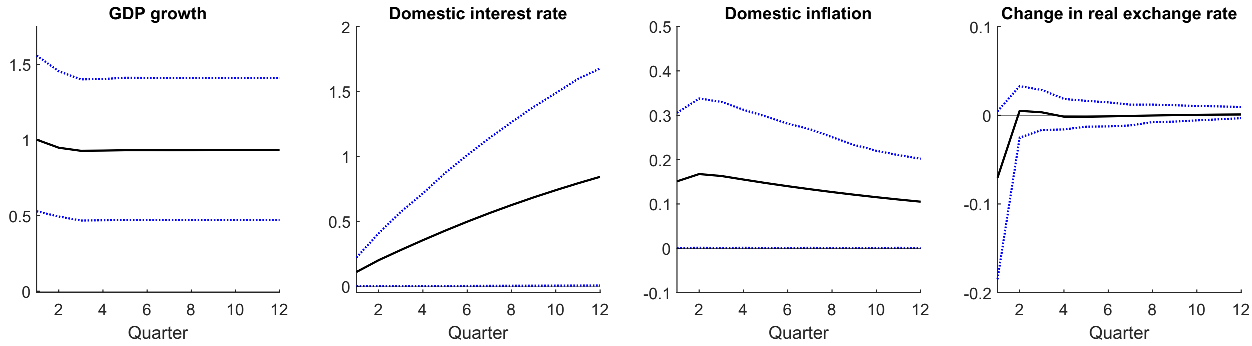

Fig. 5. Impulse response functions to a potential growth shock ($\varepsilon_t^{g^*}$).

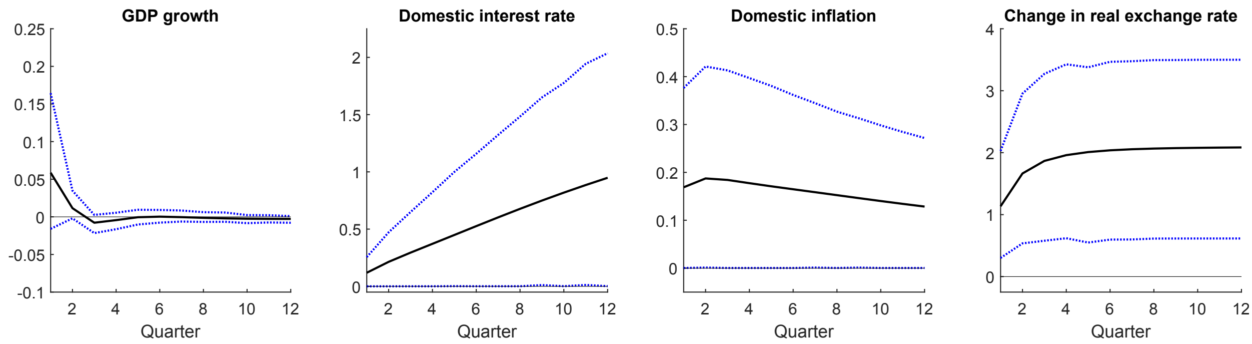

Fig. 6. Impulse response functions to an equilibrium commodity price shock ($\varepsilon_t^{\Delta p^{com,*}}$).

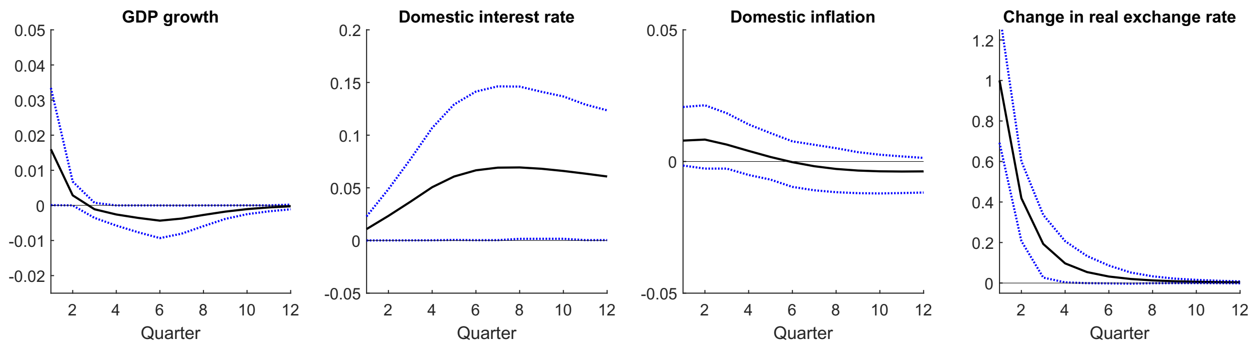

Fig. 7. Impulse response functions to an equilibrium exchange rate shock ($\varepsilon_t^{\Delta q^*}$).

high level until mid-2015 and since then sharply declined until the beginning of 2017. Since 2017, the estimated natural real rate of interest has gradually recovered and reached 3.5 percent (at the posterior mean) in the end of 2018.

To assess the stance of monetary policy based on the real interest rate gap ($r_t - r_t^*$), we also plot actual real interest rate (r_t) in Fig. 8. The monetary policy was fairly expansionary for the period 2007–2008 and 2013–2014, while it was contractionary during 2009–2010 and 2016–2017. The stance has been neutral since mid-2017 as real interest rate is within the confidence interval of natural real rate of interest. There is a sign that the stance started to become expansionary towards the end of 2018.

Fig. 9 describes the estimate of output gap from the model alongside the output gap estimated by the Hodrick-Prescot (HP) filter. There is co-movement among the two, however amplitude of the output gap is estimated to be higher in the case of HP filter. Both estimates provide consistent information about the economic condition in the sense that the HP output gap has been within the confidence interval of the model-based estimate of the output gap since 2016. The results suggest that the Mongolian economy has recovered since 2017 and is still on an expansionary path, thus building demand pressure on inflation.

Fig. 10 shows estimate for the (annualized) inflation target from the model alongside the actual annualized inflation. The estimated inflation target has been relatively stable over the whole period. Prior to 2008, the inflation target is estimated at a rate slightly below 6 percent. Around 2008, the mid-point of the inflation target increased to 6.5 percent where it has stayed for the remainder of the sample

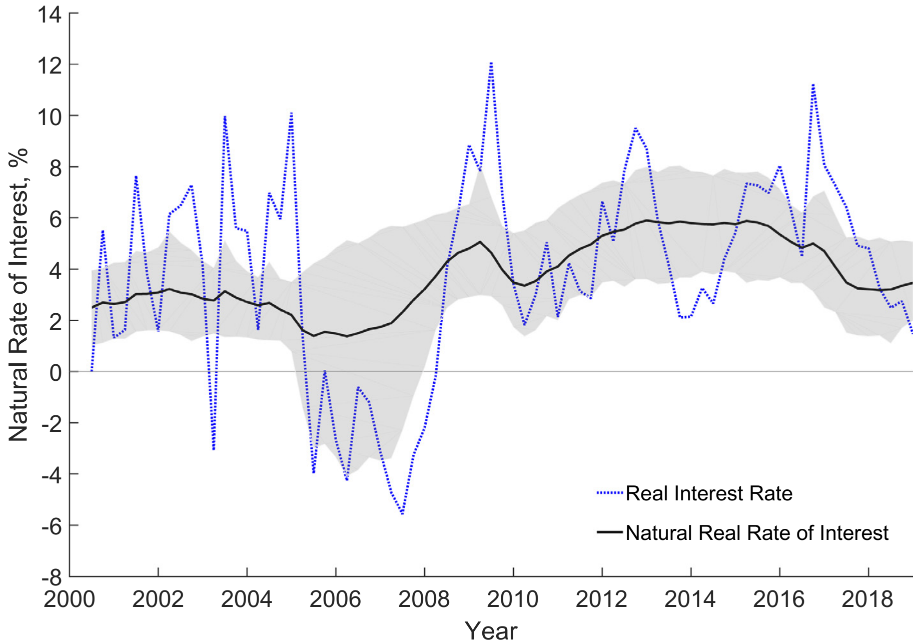

Fig. 8. Estimate of natural real rate of interest (r_t^*).

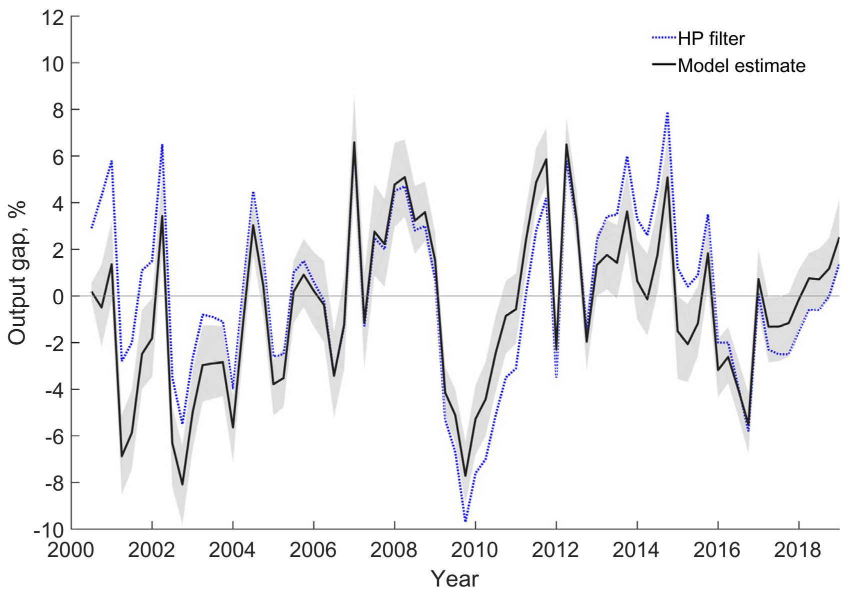

Fig. 9. Estimate of output gap ($\tilde{y}_t$). *Note*: Shaded area represents 90 percent confidence interval of estimated unobservable variables.

period. The Bank of Mongolia has not fully adopted the inflation targeting regime, and since 2012, its inflation target has been 'to keep annual inflation around 8 percent', which is closer to the top of the estimated band for inflation target. Therefore, the model estimated inflation target is in line with real-life dynamics. The natural nominal rate of interest can be found as sum of natural real rate of interest (r_t^*) and the expected inflation target ($E_t\pi_{t+1}^*$). From Figs. 8 and 10, we can see that the natural rate of nominal interest is 12–13 percent for the period 2012–2015 and around 10 percent (i.e., within the band of 8.5–11.5 percent) at the end of 2018. At the end of 2018, the policy rate was 11 percent, which lies within the band. The result also suggests that the Bank of Mongolia set its policy rate closer to the natural nominal rate of interest. Therefore, we think that the natural nominal rate of interest is high due to economic characteristics of Mongolia and it is one of key drivers of high level of interest rates in the country.

Fig. 11 presents the model estimate of the potential GDP growth along with the actual GDP growth. The estimated potential growth was gradually increasing between 2000 and 2007, however slightly deteriorated during the GFC (2008–2009). The growth accelerated

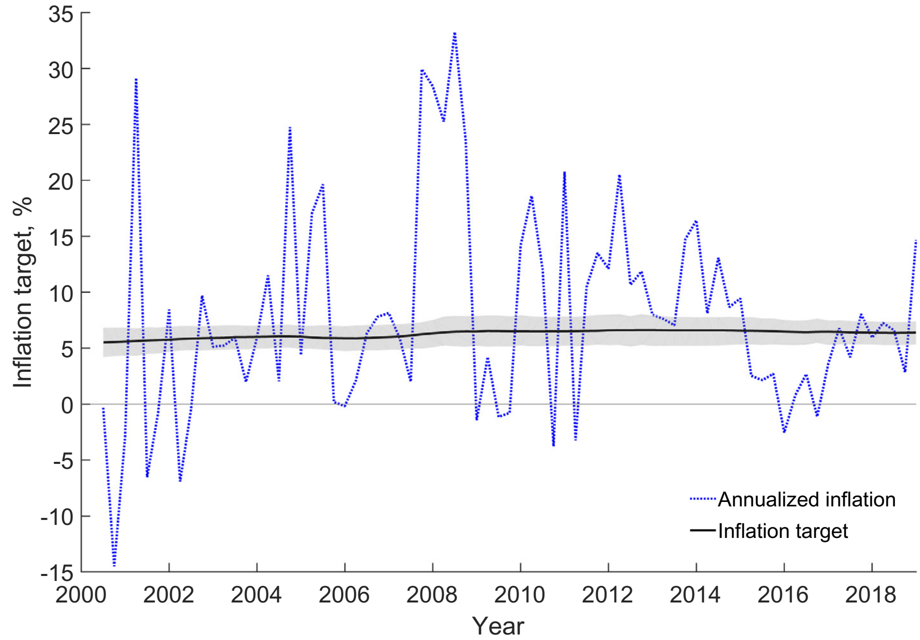

Fig. 10. Estimate of inflation target (π_t^*).

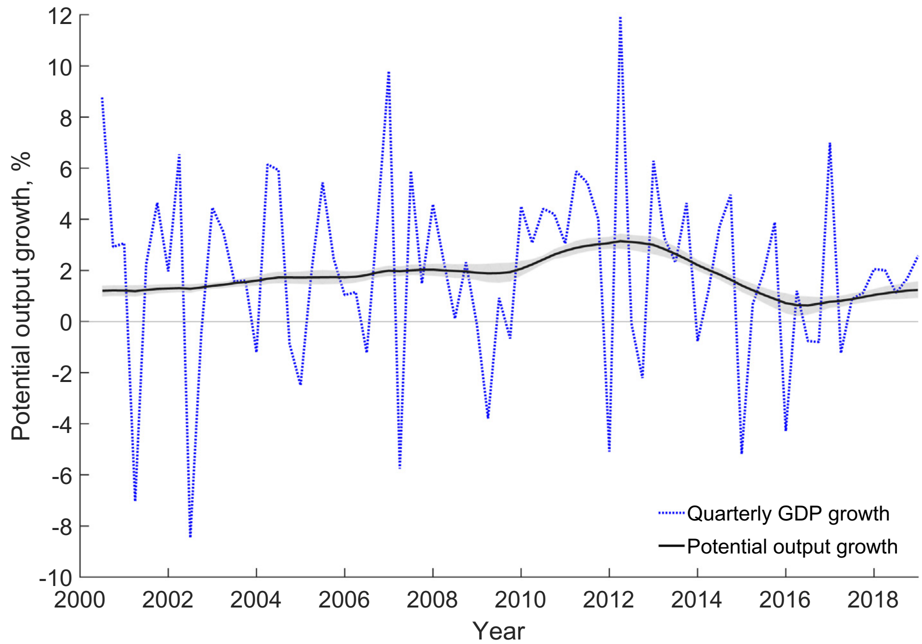

Fig. 11. Estimate of potential output growth (g_t^*). *ote*: Shaded area represents 90 percent confidence interval of estimated unobservable variables.

for the period between 2010 and 2012 since (i) commodity price and commodity demand were high, and (ii) Mongolia received a flood of FDI in the mining sector, particularly in Phase 1 of the Oyu-Tolgoi project development. In 2012, the potential growth reached 3.1 percent (equivalent to annualized growth of 12.4 percent), which was a record high growth in the last three decades. Since (i) Phase 1 of the Oyu-Tolgoi project was completed in 2012 and the second phase was delayed due to the conflict between the government and the investor (leading to sudden stop of FDI) and (ii) commodity price collapsed, the growth has sharply declined between 2013 and 2016. As of mid-2016, the growth was reduced to 0.63 percent (equivalent to annualized growth of 2.5 percent). The FDI for Phase 2 of the Oyu-Tolgoi project was started and commodity prices improved, leading to recovery in the potential growth since 2017. As of end of 2018, the growth was around 1.2 percent, 2.5 times lower than the historic record observed in 2012.

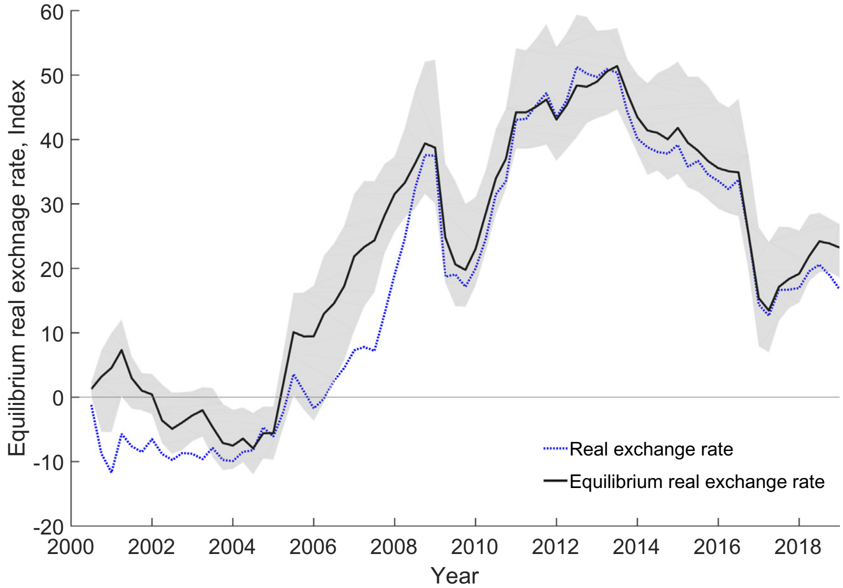

Fig. 12. Estimate of equilibrium real exchange rate (q_t^*). *Note*: Shaded area represents 90 percent confidence interval of estimated unobservable variables.

Fig. 12 illustrates the model estimate of the equilibrium real exchange rate along with the actual real exchange rate. The result suggests that for most of the period, the real exchange rate moved in line with the equilibrium real exchange rate. According to the estimation, the real exchange rate was undervalued for the period of 2000–2003 and 2007–2008 as the line of the real exchange rate is outside of the confidence interval of the equilibrium rate. There is a sign that the real exchange rate slipped to territory of underestimation in the end of 2018.

The natural real rate of interest is determined by its own lag, the annual potential output growth and annual change in the equilibrium real exchange rate (as shown in equation (1)). Fig. 13 displays the natural rate along with its determinants. From the graphical

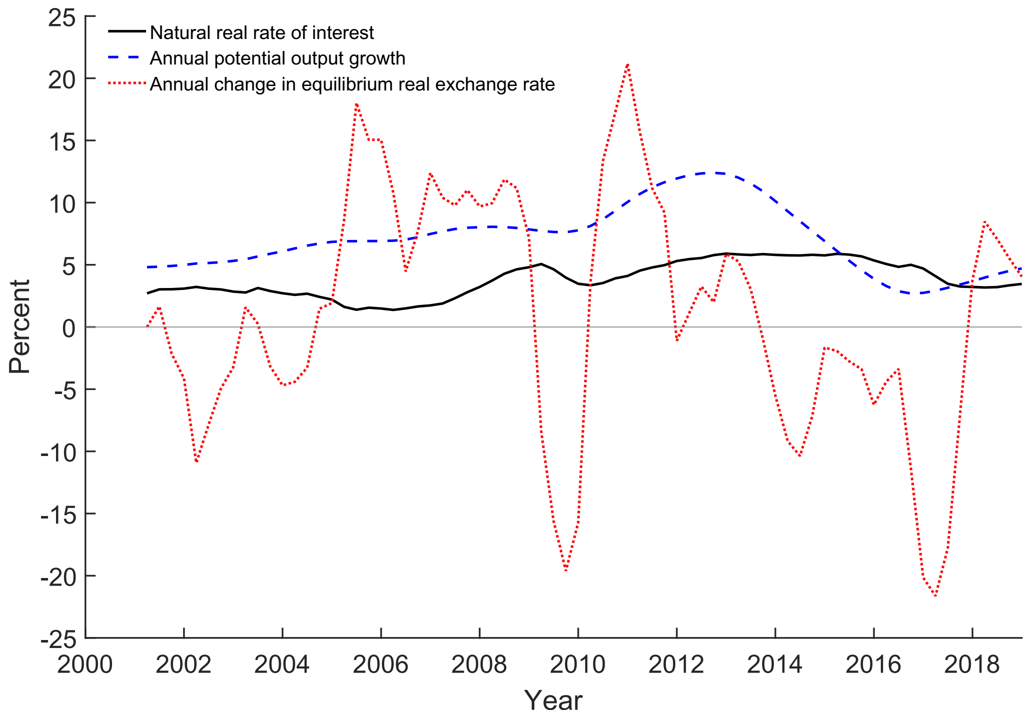

Fig. 13. Natural real rate of interest (r_t^*), annual potential growth ($y_t^* - y_{t-4}^*$) and annual change in real equilibrium exchange rate ($q_t^* - q_{t-4}^*$).

Table 3
Variance decomposition by selected shocks, in percent.

	Observed variables				Equilibrium variables	
	g_t	π_t	Δq_t	i_t	r_t^*	q_t^*
$\varepsilon_t^{r^*}$	0.3	0.1	0.3	10.6	31.7	0.0
ε_t^{y}	65.6	0.1	0.5	3.3	0.0	0.0
ε_t^{π}	0.1	95.1	0.2	0.6	0.0	0.0
ε_t^{q}	0.0	0.2	8.9	0.9	0.0	0.0
$\varepsilon_t^{\Delta q^*}$	0.1	0.0	33.4	3.2	11.2	25.2
ε_t^{i}	7.1	1.3	6.7	36.9	0.0	0.0
$\varepsilon_t^{g^*}$	12.7	0.1	0.0	7.0	18.7	0.0
$\varepsilon_t^{\pi^*}$	0.0	2.0	0.0	5.3	0.0	0.0
$\varepsilon_t^{\Delta x^{com,*}}$	0.0	0.0	1.7	0.8	1.5	3.2
$\varepsilon_t^{\Delta p^{com,*}}$	0.0	0.3	42.1	19.1	36.2	69.8
$\varepsilon_t^{\Delta fdi^*}$	0.0	0.0	0.8	0.4	0.8	1.8
$\varepsilon_t^{y,f}$	0.0	0.0	0.0	0.0	0.0	0.0
$\varepsilon_t^{\pi,f}$	0.0	0.1	1.0	0.3	0.0	0.0
$\varepsilon_t^{i,f}$	0.0	0.4	1.6	1.9	0.0	0.0
$\varepsilon_t^{\tilde{x}^{com}}$	5.7	0.2	1.2	3.9	0.0	0.0
$\varepsilon_t^{\tilde{p}^{com}}$	4.2	0.1	0.9	3.1	0.0	0.0
$\varepsilon_t^{\widetilde{fdi}}$	4.2	0.1	0.7	2.9	0.0	0.0

Notes: The numbers in the table are the posterior mean conditional variance decomposition (in percent) at period 20, which approximates (long-horizon) stationary variance (or unconditional variance).

analysis, it is evident that the natural real rate of interest in Mongolia is determined by both potential output growth and change in equilibrium real exchange rate. In particular, large deprecations in the equilibrium real exchange rate explain significant reduction in the natural rate in 2009 and 2016, when the economy was hit by adverse external shocks. The natural real rate of interest is much smoother compared to its determinants, suggesting that its own lag (source of persistence) also plays an important role in its dynamics. It is also observed that the potential growth is lower (higher) when the equilibrium exchange rate is low (high). These results suggest that the equilibrium variables may be driven by same shocks such as commodity price, commodity demand and FDI in the Mongolian economy. Such suspicion is formally examined in Section 4.3.

4.3. Variance and historical decomposition of natural real rate of interest

In this section, we examine the forecast error variance decomposition in order to investigate which shocks play important role in driving fluctuations in the natural real rate of interest, equilibrium real exchange rate and the observed variables. Since the estimated model is a richer one with several shocks and structural relations, it would be useful to assess contribution of each shock 'relative' to other shocks. Table 3 reports the conditional forecast error variance decomposition of selected variables (at period 20) evaluated at the posterior mean.

The variance decomposition analysis shows that external and demand shocks play important role in the Mongolian business cycle fluctuations. External shocks account for about 15 percent[11] of the conditional forecast error variance of GDP (at period 20) growth and 30 percent of the variance of domestic short-term interest rate. This result highlights the significance of external shocks for the Mongolian economy and is in line with the findings of Gan-Ochir and Davaajargal (2019). Shocks of domestic demand (output gap), potential growth and short-term interest rate respectively explain 65, 13 and 7 percent of the variances of GDP growth.

Domestic inflation is mainly driven by supply-driven shocks. Demand factors explain less than 5 percent of the inflation variance. This result is in line with the fact that the inflation volatility is led by supply-side factors (i.e., Gan-Ochir and Undral, 2018).

In case of Mongolian economy, high level of policy rate and fluctuations in nominal exchange rate are associated with its structural vulnerability to external shocks. Shocks of equilibrium commodity price and equilibrium exchange rate account for 42 and 33 percent of the variances of change in real exchange rate fluctuations, while 16 percent of the variance is explained by shocks of the domestic interest rate and real exchange. This result implies that the real exchange rate serves as a shock absorber, which helps to stabilize the domestic economy in the wake of external shocks. External shocks jointly account for more than 30 percent of the variances in short-term interest rate. About 25 percent of the interest rate variance is explained by shocks to domestic equilibrium variables ($\varepsilon_t^{r^*}$, $\varepsilon_t^{\Delta q^*}$, $\varepsilon_t^{g^*}$, $\varepsilon_t^{\pi^*}$), while the monetary policy shock accounts for 37 percent of the interest rate fluctuations. The results indicate that the monetary policy (short-term interest rate) has been responsive to changes in the global market and the domestic economy.

Fluctuations in the natural real rate of interest are mainly explained by external shocks in Mongolia, a commodity-dependent

[11] The external shocks account for 22 percent of conditional variance decomposition of GDP growth (at period 1), suggesting that the shocks play very important role in short run business cycle fluctuations.

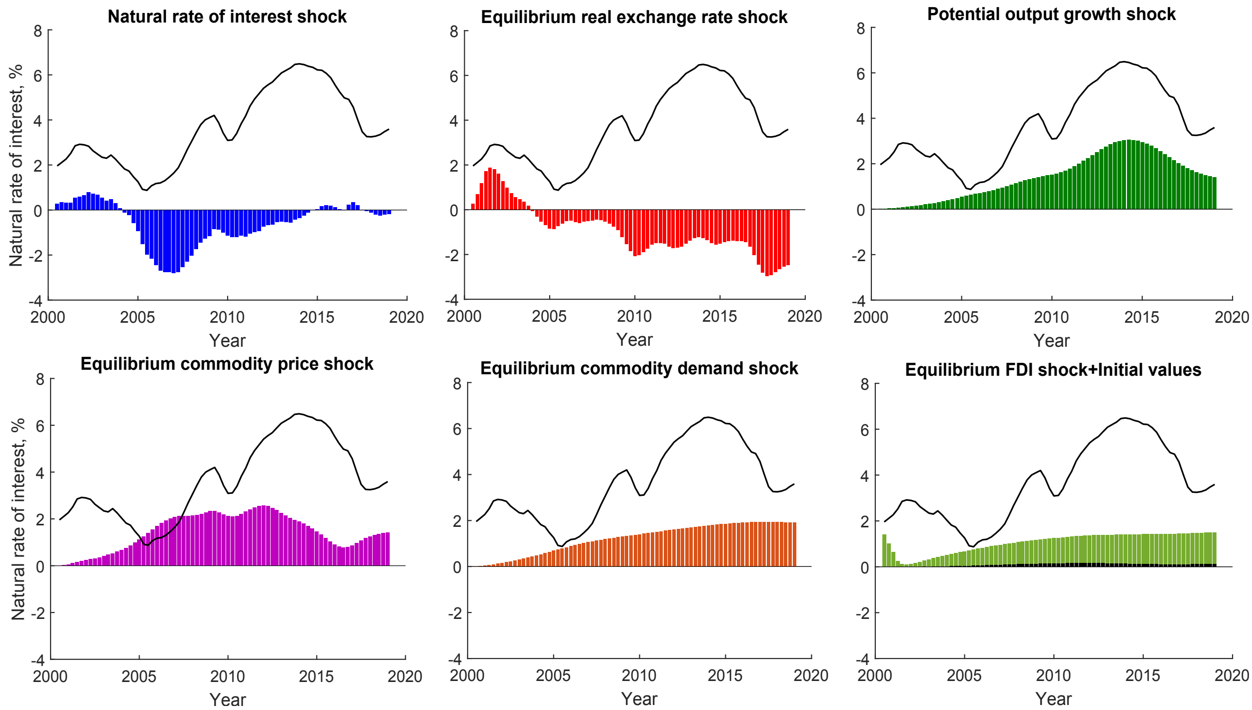

Fig. 14. Historical decomposition of natural real rate of interest: Contribution of shocks. *Notes:* The numbers in the plot are posterior mean estimate of the natural real rate of interest and posterior mean contributions of shocks. In the last plot of the figure, the black bar and the light green bar represent the contributions of equilibrium FDI shock and initial values, respectively.

economy. For example, shocks of equilibrium commodity price, equilibrium commodity export demand and equilibrium FDI jointly account for about 40 percent of the fluctuations in natural rate. In line with empirically viable assumption described in equation (1), shocks of potential growth and equilibrium real exchange rate account for 18.7 and 11.2 percent of the fluctuations in natural rate, respectively. Its own shock explains 31.7 percent of its variance. The external shocks mainly transmit through the equilibrium real exchange rate, as the same shocks account for 75 percent of fluctuations in the equilibrium real exchange rate.

Overall, external shocks (i.e., shocks of equilibrium commodity price, equilibrium commodity export demand and equilibrium FDI) play crucial role in the economy. The analysis suggest that the transmission mechanism is as follows: the external shocks are initially reflected in the equilibrium exchange rate, which affects the natural real rate of interest and real exchange rate. Then the short-term interest rate follows the change in natural real rate of interest. Both short-term interest rate and real exchange rate in turn affect domestic macroeconomic variables. Moreover, monetary policy has been an active stabilizer of the economy, which is frequently hit by external and domestic shocks.

It is also important to use the estimated model to analyze historical decomposition, which describes how these shocks have shaped the natural rate of interest over time. Fig. 14 displays historical decomposition of natural real rate of interest over the period 2000:Q2-2018:Q4. For the period 2000–2004, changes in natural real rate of interest are mainly determined by equilibrium real exchange rate and its own shocks. It is in line with the fact that the Mongolian economy was relatively less dependent on the mining sector for the period. Therefore, the decline in natural rate between 2001 and 2012 is mainly driven by negative shocks to both natural real rate of interest and real exchange rate.

Though the shocks of natural real rate of interest and real exchange rate remained negative, the natural real rate of interest was raised during 2005–2012 due to positive impact of shocks in potential growth, equilibrium commodity price and equilibrium commodity demand. In particular, shock of equilibrium commodity price plays an important role for the acceleration during 2005–2008, while rising natural real rate for during 2010–2012 is determined by all three shocks. The temporary reduction in 2009 (during the GFC) is mainly associated with shocks of natural real rate of interest, equilibrium real exchange rate, and equilibrium commodity price.

As super cycle of the commodity market ended in 2012, natural real rate of interest started to decline until 2017, mainly driven by shocks of equilibrium commodity price, equilibrium real exchange rate and potential growth. Though contribution of potential growth has been decreasing, natural real rate of interest is starting to increase from 2017, thanks to positive shocks of equilibrium commodity price and commodity demand. There is evidence that the contribution of potential output growth and the natural real rate of interest have strong co-movements over time. Impact of equilibrium FDI shock on the natural real rate of interest is estimated to be weak.

5. Conclusion

In this paper, we estimated the natural real rate of interest, inflation target, output gap, potential output growth, and equilibrium real exchange rate for Mongolia, a commodity exporting economy, using a small open economy New Keynesian model with time-varying equilibrium variables. The model is estimated using Bayesian methods on the Mongolian data over the period of 2000Q1-2018Q4, and the dynamics of the unobservable variables within the estimated model are extracted using the Kalman smoother.

We find evidence of transitory shocks to the natural real rate of interest, supporting the result shown by Lewis and Vazquez-Grande (2018) - the natural real rate of interest is prone to both permanent and transitory shocks. The estimated model is able to replicate the stylized facts of the impulse responses, suggesting that the shocks are properly identified.

The natural real rate of interest is procyclical in the Mongolian economy. Since the collapse of the commodity super cycle in 2012, the estimated natural rate of interest fell sharply. Since 2017 it has been gradually recovering and reached 3.5 percent (at the posterior mean) in the end of 2018. The stance of monetary policy has been neutral since the second half of 2017 as real interest rate is within the confidence interval of the estimated natural real rate of interest. The natural nominal rate of interest remains at double-digits due to the economy's characteristics and it has been one of key drivers of high interest rates in Mongolia.

The estimated output gap suggests that the Mongolian economy has recovered since 2017 and is still in the expansion path, building demand pressures on inflation. The estimated inflation target has been relatively stable at around 6–6.5 percent (at the posterior mean) over the sample period. However, there has been noticeable variation in the potential output growth and equilibrium real exchange rate over the sample period. The estimated potential growth accelerated during 2010–2012, and reached 3.1 percent (equivalent to annualized growth of 12.4 percent), which was a record high growth within the last three decades. The growth has sharply declined between 2013 and 2016, reaching 0.63 percent (equivalent to annualized growth of 2.5 percent) in mid-2016, and has recovered since 2017. For most of the sample period, the real exchange rate moved in line with the equilibrium real exchange rate. There is a sign that the real exchange rate was undervalued during 2000–2003 and 2007–2008.

Our results show that the natural real rate of interest in Mongolia is determined by both potential output growth and change in equilibrium real exchange rate. In particular, external shocks play an important role as drivers of the natural real rates of interest. For example, shocks of equilibrium commodity price, equilibrium demand for commodity exports and equilibrium FDI jointly account for about 40 percent of the natural rate fluctuations. The external shocks mainly transmit through the equilibrium real exchange rate as the shocks account for 75 percent of the equilibrium real exchange rate fluctuations. Shocks of potential growth and equilibrium real exchange rate account for 18.7 and 11.2 percent of fluctuations in the natural rate, respectively. Since 2004, shocks of natural real rate of interest and real exchange rate shocks were negative, while shocks of potential output growth, equilibrium commodity price and equilibrium commodity demand were positive. The contribution of potential output growth and the natural real rate of interest have strong co-movements over the sample period, and the recent rise in the natural rate is mainly driven by positive shocks of equilibrium commodity price and commodity demand.

These results provide some implications. First, it is important to explicitly incorporate external factors (demand for commodity export, commodity price, and FDI) into the model when measuring the natural rate of interest for a commodity-exporting economy. Second, as the economy is vulnerable to external shocks, it is essential to build fiscal and foreign exchange buffers, to introduce measures to mitigate the boom-bust cycle and to ensure continued sustainable and inclusive growth. Finally, forward-looking, counter-cyclical monetary and fiscal policies would have smoothening effect on the amplitude of macroeconomic volatility driven by external shocks. Consequently, such policies have strong implications on maintaining favorable macroeconomic environment, which is a necessary condition to lower the high interest rates.

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