***Provisional translation***

**ONE. GENERAL PROVISION**

The purpose of this guideline is to give knowledge of general statistical and mathematical modeling tools that are used to calculate operational risk to bank employees and supervisors.

**TWO. RISK ESTIMATION PRINCIPLE**

All models that included in this guideline are based on conditional (Bayesian) and unconditional (Frequentist) probability theory and philosophy. When choosing the usage of above principles, banks and financial institutions shall consider their operations’ uniqueness.

**THREE UNCONDITIONAL PROBABILITY MODELS**

**3.1.Risk estimation principle**

It is common to use random discrete variables in calculating operational risk. Thus it is very important to pay attention to the frequency of the data and the expected loss when calculating operational risk.

Primary principle of calculating operational risk using unconditional probability is to find probability distribution of the random variable that is consistent with loss data. Commonly applied probability distributions for risk estimations are included in the Annex 1 of this guideline. Review of consistency of loss data with the probability distribution of random variable is evaluated according to 3.2 of this guideline.

**3.2. Choosing a probability distribution for the risk estimation**

Following tests statistics show the degree of consistency of loss data with common probability distributions:

Kolmogorov*–*Smirnov test: The main idea is to compare empiric and researcher’s chosen probability distribution and to calculate distance of two functions. The main result of this test is KS(Dn) and is calculated as shown below:



Here: *Dn* is the test statistic, *n* number of observations, *Fn(x)*=(n-k+0.5)/2, *k* is data input rank, *F(x)* is distribution function.

Following table shows the function’s critical values with the corresponding different significance levels:

|  |  |
| --- | --- |
| Limits | Significance level |
| 1,07\*[n-1/2] | 20% |
| 1,22\*[n-1/2] | 10% |
| 1,36\*[n-1/2] | 5% |
| 1,63\*[n-1/2] | 1% |

Kolmogorov*–*Smirnov test is deemed statistically weak because it chooses the maximum value of distance of associated functions. Disadvantages of Kolmogorov-Smirnov test is more noticeable in samples with small number of observations.

Anderson-Darling test: This test is advanced version of the Kolmogorov-Smirnov test. Following method is used to calculate the test statistics.



Here:



n- is the number of observations, *F(x)* is researcher’s chosen probability distribution function, *f(x)* is probability distribution density function where *Fn(x)* =(n-k+0.5)/2. Following are critical values of test statistics:

|  |  |  |
| --- | --- | --- |
| Size | At 5% significance level | At 1% significance level |
| 1+0,2\*[n-1/2] | 0.757 | 0.05 |
| 1+0,3\*[n-1/2] | 1.321 | 0.01 |

This test statistic is more robust than Kolmogorov-Smirnov test in that it considers all possible distances calculated across vertical axis, with a help of Ψ function distribution variance is neutralized, also with a help of distribution density, the difference between the functions are weighed with their corresponding probabilities.

Kramer Fon Mises test: It is a statistical test that calculates distance between the functions, from which the square average is derived and compared. The indicator is calculated as follows:



Kramer Fon Mizes test is unique in that the test is adjusted with its number of observations. Following are critical values of test statistics:

|  |  |  |
| --- | --- | --- |
| Size | At 5% significance level | At 1% significance level |
| 1+0,2\*[n-1/2] | 0.124 | 0.174 |
| 1+0,16\*[n-1/2] | 0.222 | 0.338 |

**3.3.Risk estimation using extreme value distribution**

It is common knowledge that operational risk data is limited; also it is not effective to use normal distribution and other similar distributions due to the fact that value of loss is very high if loss event is occurred. Thus researchers, in that situation, prefer to use distribution of extreme values rather than using whole distribution. “*Frechet*”, “*Gumbel*”, and “*Weibull*” distributions are frontrunners of extreme value distribution.

Since extreme value distribution and its similar theories cover broad range of topics, it is better to present the core ideas and summaries.

Theoretical part: For distribution function *FR* defined in interval (*l*, *u*), with random values, the extreme values *Zn* is defined as follows:



If sample values in *FR* distribution are independent, distribution of random values *Zn* can be shown as follows:



Due to the problems that occur by using formula (1.6) directly, the nomralization is incorporated with parameters αn indicating scope and βn indicating location to the formula. The formula after normalization is shown below:



where “*τ* “ is called the index indicating the “tail” of the distribution. If (*τ* >0), then (1.7) is of “*Weibull*”, if (*τ* =0) then (1.7) is of “*Gumbel*” and if (*τ* <0) then (1.7) is of “*Frechet*” shape of distribution.

VaR risk calculation: Building on the theoretical part of this guideline and extreme value theory, the value at risk (VAR) concerning the operational risk of an institution at a certain significance level can be assessed as:



By manipulating (1.8), we can calculate the unexpected loss as follows:



Parameter estimation: As there are many ways to calculate parameters of (1.9), let us apply the moment method of (1, 2, 3 ….)th order. If we denote the moment of order “*r*” as “*mr*”, for “*n*” number of observation and *Xj* random variable the moment is defined as follows:



Where *Uj* is independent from *F(Xi)* distribution and is defined by formula *p*j*=(n-j+0.5)/n.* According to Hoskings’ calculation (Hosking 1985), distribution moments can be represented using associated parameters:



Iteration method is used for parameter estimation based on (1.11) which in turn brings numerous problems. Thus in order to simplify the complexity, Hosking proposed the following formulas to calculate parameters:









Example: The table below shows the amount of missing cash assets or the assets at the branch of Bank A due to the illicit activities (fraud):

|  |  |
| --- | --- |
| Year | Deficit |
| 1992 | 250,000.00 |
| 1993 | 201,600.00 |
| 1994 | 199,000.00 |
| 1995 | 182,000.00 |
| 1996 | 182,000.00 |
| 1997 | 175,000.00 |
| 1998 | 165,200.00 |
| 1999 | 160,300.00 |
| 2000 | 150,000.00 |
| 2001 | 110,000.00 |
| 2002 | 100,000.00 |
| 2003 | 95,000.00 |

Based on the information above, the parameters of interests and the amount of operational risk are:



Based on value of parameters, the unexpected loss from the operational risk equals

VAR =204,161.91. We used *pext*=0.01 for the loss calculation.

**3.4.Risk model, calculation validation**

There are many methods to validate risk model used in risk estimation with actual outcomes. Of these methods, the simplest and widely used one is calculating the share of number of excess of actual loss over the projected loss. In other words, in actuality there are instances where the actual loss is significantly higher than the projected loss using mathematical and statistical tools raising the question to modify or change the existing model.

Besides simple ratios, there are many modern approaches that are based on statistical theories. For the purpose of informing, Kupier test, included in Basel committee documents, is introduced below.

As the test statistic states for “*T*” number of observations, “*V*” number of occurrences that actual loss exceeded the projected loss, the associated probability equals . If null hypothesis is *Ho: p=p\** (here p\* is significance level) test statistics is defined as below:



Here *LRUC* has χ2(1) distribution and if *LRUC*>3.84 hypothesis is cancelled. In other words if *LRUC*<3.84, then the model has to be changed or modified.

**FOUR. CONDITIONAL PROBABILTY MODELS**

**4.1.Background**

It is common to have few or nonexistent quantitative data available for operational risk estimation. In those cases it is only recommended to use simulation method to derive quantitative data. Bayesian statistics calculation widely uses simulations based on conditional probability, and this makes it very suitable for operational risk estimation.

One drawback to Bayesian statistics calculation is that it considers many subjective factors, which makes it questionable for some researchers. Such disagreements usually caused by different use of statistical terms, and goal of statistics is not clearly defined. For example, many people argue on the term “parameter distribution” and “probability distribution.”

**4.2.Theoretical part**

Let’s suppose statistics model is aimed to define states of vector  that consist of random variables with  dimension. Here the notation “t” depending on condition it can define time, space, observation unit and etc. Let’s suppose  shows the first t observation of the sample. Also *ψ is*’s sample set, where **Y***t*’s sample set is *Ψ* and . In this case “*A*” statistical model can represent a sequence of density functions in following way:



Here θA is  dimensional vector which contains unknown random variables where has to be satisfied. The specification of θA- depends on the choice of the model. For example, for the most econometric models, θA is unknown parameters or latent variable.

*p*( . ) is *v*( . ) dimensional density function. The dimension v( . ) determines whether the random variable is discrete or continuous.

Using condition of the model “*A*” model and θA unknown vector, we can define density of the **Y**t:



If, for a given model, there is no relation between  vectors, then;



In this case it is:



As (1.18) and (1.20) only defines part of the model, we need to take into account the concept of the prior density to get the general picture of the model. Prior density p(θA|*A*) is the set of quantifiable numbers that belongs to θA vector consistent with the “*A*” model. In other words, p(θA|*A*) containes θA ‘s information before the information “**y”** is revealed.

By combining prior density and (1.18) we can get “y” and θA’s general density equation:



At the same time we can show general density in a following way



Thus by combining (1.21) and (1.22) we get:



In reality, most of the time it is difficult to estimate *p*(*y*|A) and if the general form of p(θA|y,A) can be derived it is only possible to make conclusion Thus it is recommended to use following simplified formula:



Here p(θA |y,A)’s density is posterior density. In conclusion the purpose of calculating p(θA |y,A) is to reach the unambiguous conclusion on a given unknown variable by combining one’s prior knowledge with the number of observation.

**4.3.Using Bayesian theory for risk estimation**

For the most risk modeling, mean and deviation are of importance for conducting researches and analysis. However, in reality, due to various reasons it is difficult to estimate these parameters thoroughly.

For example, when calculating foreign currency risk using VaR method, exchange rate fluctuation (unknown parameter) should be calculated. However factors such as political instability, economic cycles, exchange rate regime, balance of payments, foreign currency reserve, the outcome and the expectation for the foreign currency volatility can differ. In other words, the final amount of loss is dependent on the chosen value for the foreign exchange volatility.

This uncertainty causes difficulties for bank and financial institution management to make prudent decisions on risks. For example: If a bank is to place capital to absorb the loss from operation risks, it is difficult for banks to precisely determine the amount of loss due to the uncertainty of the parameters

These statistical methods that are based on Bayesian theory tackle those problems very well. This can be seen from the following formula:



Here: from previous example, *ω* is an expected loss from exchange rate fluctuations, θA is currency volatility.

**4.4. Choosing prior density**

In Bayesian methods, final outcome depends on prior distribution choice. Thus, let us discuss more about prior density.

One of the most commonly used prior density is “elicited prior”. In most cases it relates to the predictions of experienced experts in a chosen field.

To determine unknown parameter’s expected values, so-called “*uninformed prior*” is important. An example of this is distribution of random values with the same probability in (0,1) interval. As regards *“Uninformed prior”* we shall discuss about “*Jeffreys prior*”-, because “*Jeffreys prior*” tends to maintain its inherent characteristics even after some changes were made.

“Conjugate prior” is a prior density which has similar function shape with (1.18) and (1.20).

A prior density with some missing feature is called “*hyperprior*”. “*Hyperpriors*” can be used as a combination, however we shall note that this raises uncertainty.

**4.5. Bayesian sample method and parameter estimation**

With the recent advancements in technologies and information processing, it is made possible to actually calculate many Bayesian-based different methods. For purpose of examples Markov Chain Monte Carlo (MCMC) methods are used below.

Models that based on Bayesian theories try to evaluate following formula:



Here π(x) is posterior distribution. Monte Carlo method tries to calculate E[f(x)] by sampling specified numbers for π(x).



However, since in reality it is common for π(x) to be of non-standard distribution form, f it is limited to sample numbers. The most common way of overcoming this problem is to use Markov Chain series.

Gibbs sampler: It divides θA parameter into following subsets:



Where for any θ*b* vector:

Similar to this

Considering (1.29) and (1.30) let’s put following formula:



For Gibbs method it is crucial when choosing number of subsets in (1.28), it should be possible to sample density function through the conditional probability formula - .When this criteria is met, the initial sample value θ(0) is chosen from distribution density p(*θ* | *I*) in a following way:



Next step is



Here. Following this principle with m steps



These steps forms {θ(m)} Markov chain and migration probability is formularized as follows:



Metropolis-Hastings Algorithm To define model parameters with a help of this algorithm, any migration probability density function *q*(*θ*\* | *θ*, H), the argument *θ*\* that is the value indicating density, initial values that needed to start the calculation must be given beforehand. In each step of the algorithm, the test whether the sample value *θ*\* from  is compatible to θ*(m)* should be performed. This can be determined with following probability:



If the sample value *θ*\* is incompatible with θ*(m*, it is inferred as θ*(m)* = θ*(m-1)*

To further elaborate the algorithm, the kernel (any non-negative, proportional to the given density value) of the “approved” sample values of distribution density are defined as follows:



Also, the probability to reject the sample values is calculated as:



In other words, for any given θ, (1.38) is able to shows the probability that θ\* is rejected and the probability can be determined before sampling the θ\*.

Thus, Markov chain related to this algorithm is extracted from migration probabilities defined at “v” dimensional sets -  in a following way:



Therefore, to define migration density, Dirac delta function is calculated in a following way:



Based on this, θ(m-1) indexed Markov chain is defined as follows:



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